Math 262 - 9 September 2019

Warm-Up: For each of the following experiments, state the sample space and any three events:

outcome that heads never appears

(a) A coin is flipped until heads appears, and the number of flips is recorded.

$S = \{H, TH, TTH, TTTH, \ldots\} \cup \{T\ldots\}$

EVENTS: $\{TH\}$ $\{1,2,3,4,5,6\}$ $\{2,4,6,8,\ldots\}$

or

$S = \{T^iH \mid i \geq 0 \text{ integer}\} \cup \{T^\infty\}$

or

$S = \{1,2,3,4,\ldots\} \cup \{\infty\}$

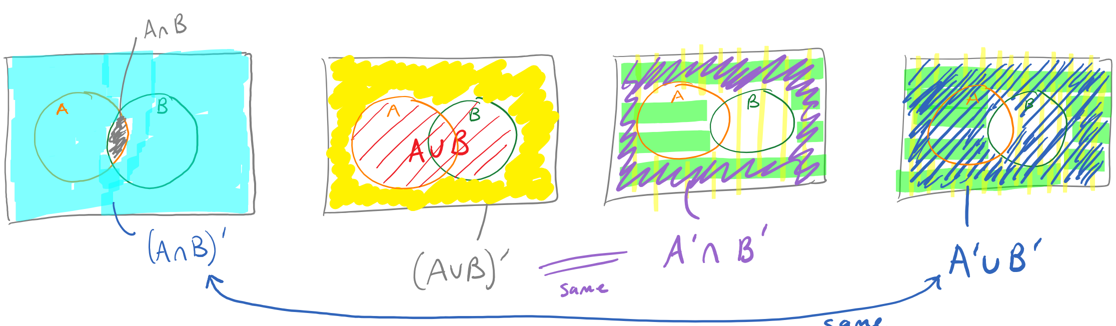
(b) A real number is selected between 0 and 1.

$S = (0,1)$ open interval

EVENTS: $(0, \frac{1}{2})$ $\{0.1\}$ $\{0.1, 0.9\}$

1. Let A and B be events in a sample space. Draw Venn diagrams to illustrate each of the following events:

$$(A \cap B)' \qquad (A \cup B)' \qquad A' \cap B' \qquad A' \cup B'$$



What do you observe from your Venn diagrams?

DE MORGAN'S LAW:

$(A \cap B)' = A' \cup B'$ ⟵ the complement of an intersection is the union of the complements

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AXIOMS OF PROBABILITY

1. $P(A) \geq 0$ for any event A

2. $P(S) = 1$ Probability that something happens is 1.

3. If $A_1, A_2, A_3, \ldots$ is an infinite collection of disjoint events, then $P(A_1 \cup A_2 \cup A_3 \cup \cdots) = \sum_{i=1}^{\infty} P(A_i)$

Prob. of union of disjoint events is sum of the probabilities

2. Show that $P(\emptyset) = 0$ follows from Axiom 3.

Let $A_1 = A_2 = A_3 = \cdots = \emptyset$. That is, $A_i = \emptyset$ for all i

Axiom 3: $P(\emptyset \cup \emptyset \cup \emptyset \cup \cdots) = \sum_{i=1}^{\infty} P(\emptyset)$

$$P(\emptyset) = \sum_{i=1}^{\infty} P(\emptyset)$$

This only makes sense if $P(\emptyset) = 0$.

COMPLEMENT RULE: For any event A, $P(A') = 1 - P(A)$.

Why? Let $A_1 = A$, $A_2 = A'$, and $A_3 = A_4 = A_5 = \cdots = \emptyset$

Axiom 3: $P(A \cup A' \cup \emptyset \cup \emptyset \cup \cdots) = P(A) + P(A') + \sum_{i=3}^{\infty} P(\emptyset)$

$$P(A \cup A') = P(A) + P(A') \qquad 0$$

$$1 = P(S) = P(A) + P(A')$$

Thus, $P(A') = 1 - P(A)$.

3. Show how the Complement Rule implies that $P(A) \le 1$ for any event A.

Complement Rule: $1 = P(A) + P(A')$

Recall Axiom 1: $P(A') \ge 0$

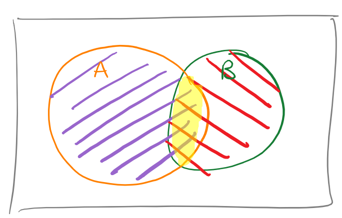
Then: $P(A) = 1 - P(A')$ is one subtract something non-negative

So: $P(A) \le 1$

4. If A and B are disjoint, Axiom 3 says $P(A \cup B) = P(A) + P(B)$. How could you modify this statement to hold if A and B are not disjoint?

Let $A_1 = A$, $A_2 = B$, $A_i = \emptyset$ for $i \ge 3$.

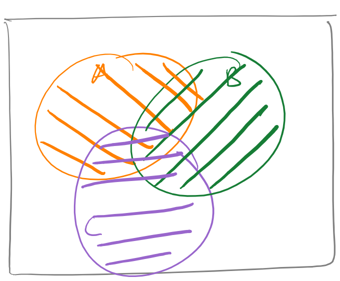
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



ADDITION RULE

$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

5. Generalize your answer from #4 above to three sets. That is, what can you say about $P(A \cup B \cup C)$?



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

ADDITIVITY PROPERTY

or INCLUSION-EXCLUSION PRINCIPLE