COUNTING PRINCIPLES

Ways of choosing k items from n:

|	ORDER IMPORTANT	ORDER NOT IMPORTANT
WITH REPLACEMENT	n^k	$\binom{k+n-1}{k} = \frac{(k+n-1)!}{(n-1)!\,k!}$
WITHOUT REPLACEMENT	$_nP_k = \frac{n!}{(n-k)!}$	$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$

CASE IV: Order not important, choosing with replacement
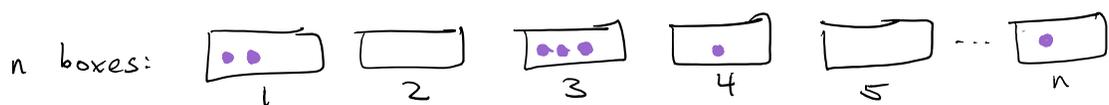
examples: Yahtzee dice; making a fruit smoothie;

assigning jobs to people, if one person can do multiple jobs
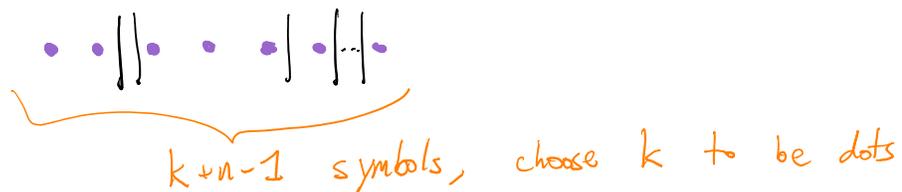
EQUIVALENT PROBLEMS:

① Selecting k items from n, with replacement, order not important

n items:

k selections
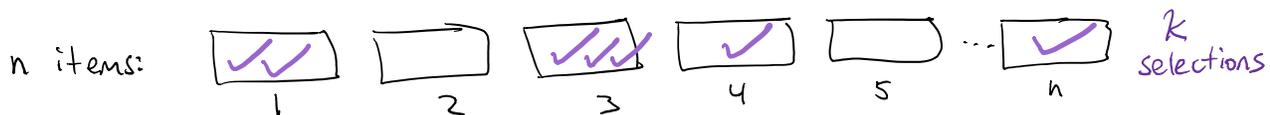
② Placing k unlabeled balls into n labeled boxes.

n boxes:

③ Ordering k dots and $n-1$ lines

$k+n-1$ symbols, choose k to be dots

$\underline{1} \cdot \cdot \cdot \cdot \underline{1} \cdot \underline{1} \cdot$ choose k to be dots

$\underbrace{\qquad\qquad\qquad}_{k+n-1 \text{ blanks}}$

How many ways to choose k out of $k+n-1$?

$$\binom{k+n-1}{k} = \frac{(k+n-1)!}{(n-1)!\, k!}$$

This is the answer for all 3 equivalent problems above!