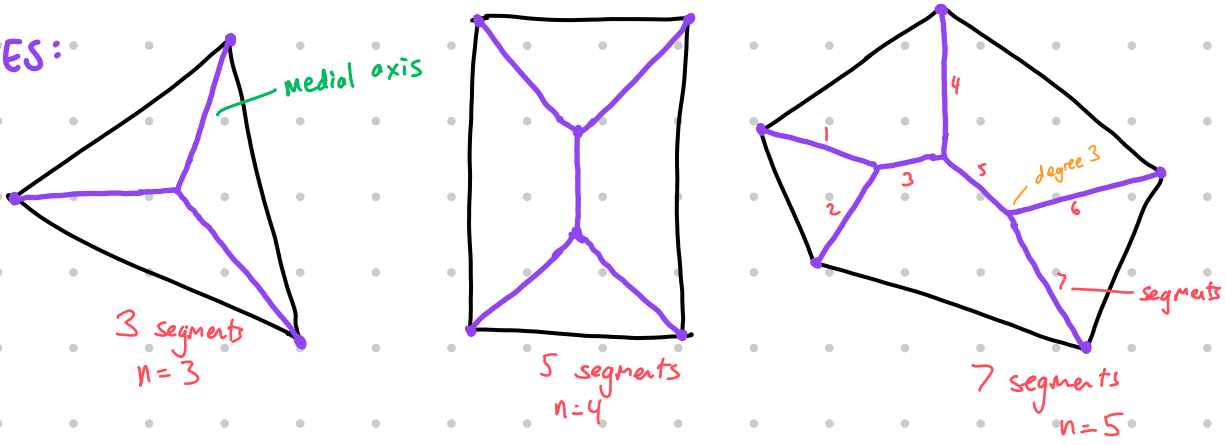
21 January 2025

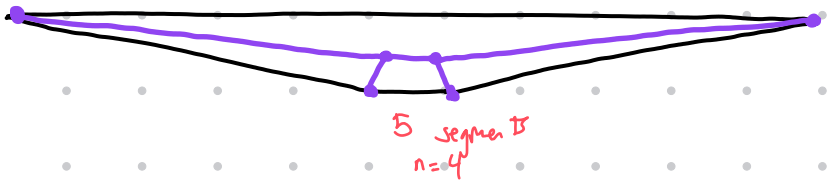
RECALL: Voronoi edges consist of points that have 2 (or more) closest sites

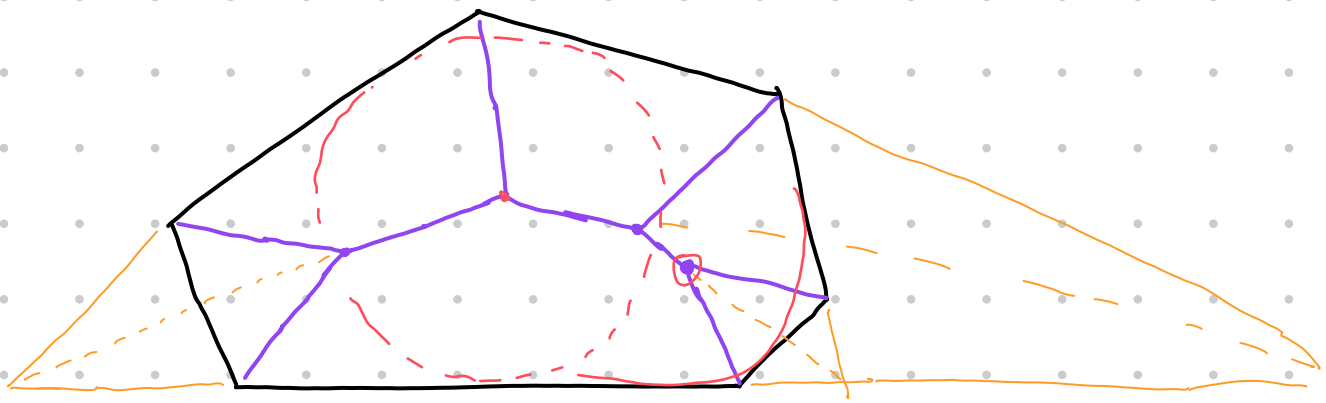
MEDIAL AXIS

For a polygon P, the **MEDIAL AXIS** (or **CUT LOCUS**) M(P) is the closure of the set of points that have two or more closest points in the boundary of P.

EXAMPLES:





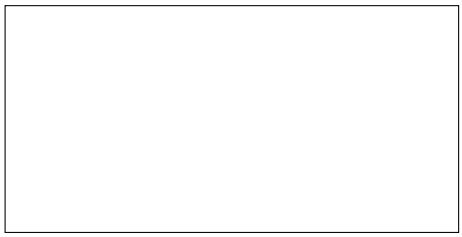


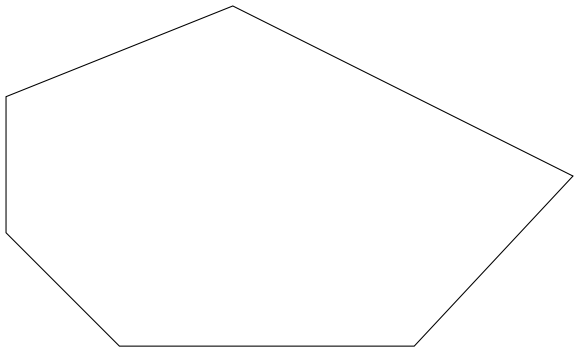
Medial Axis

MATH 261 Computational Geometry

Part 1: Convex Polygons

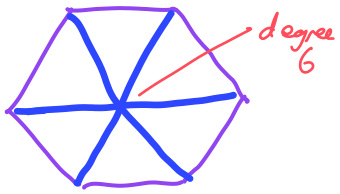
1. Sketch the medial axis for each of the following polygons.





2. Let P be a convex polygon of with n vertices. Is it possible for $M(P)$ to have a vertex of degree n? Either give an example or explain why this is not possible.

Yes, let P be a regular n-gon



3. Let P be a convex polygon with n vertices. What is the maximum and minimum number of segments of the medial axis $M(P)$?

minimum: n (eg. regular n-gon)

maximum: 2n-3

4. Design an algorithm that finds the medial axis of a convex polygon. Be as specific as you can about the implementation details. What is the computational complexity of your algorithm?

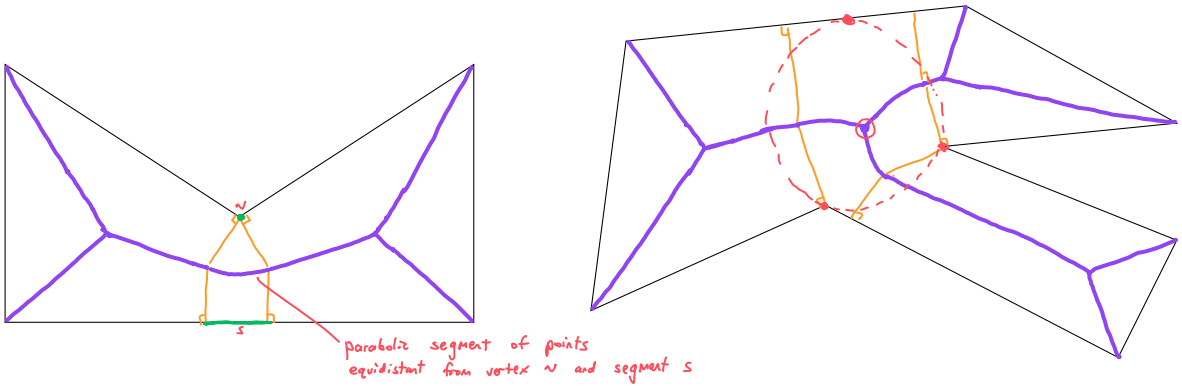
Let P be a convex n-gon. Assume general position of vertices. (no parallel sides)

- Construct angle bisectors from vertices of P.
- Maintain a priority queue of intersections, prioritized by radius of the circle centered at the intersection point and tangent to 3 edges of P
- For each intersection in the queue: extend edges of P, construct new angle bisector, and add new intersections to the queue
- The final 3 bisectors intersect at the center of the inscribed circle of a triangle.

complexity: $O(n \cdot \log(n))$

Part 2: Nonconvex Polygons

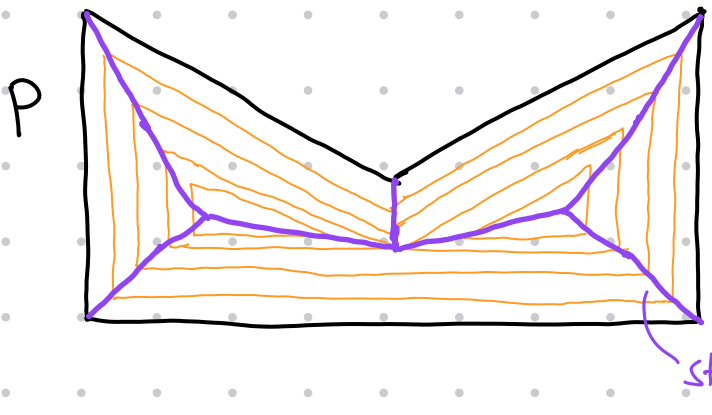
5. Sketch the medial axis of each of the following polygons.



6. Is there a nonconvex polygon whose medial axis consists entirely of straight segments? Either give an example or explain why this is not possible.

7. What is the minimum number of segments in the medial axis for an arbitrary polygon of n vertices?

STRAIGHT SKELETON



Construct offsets of polygon, inside the polygon, with edges parallel to the original edges.

Trace out the paths of vertices as the offsets shrink.

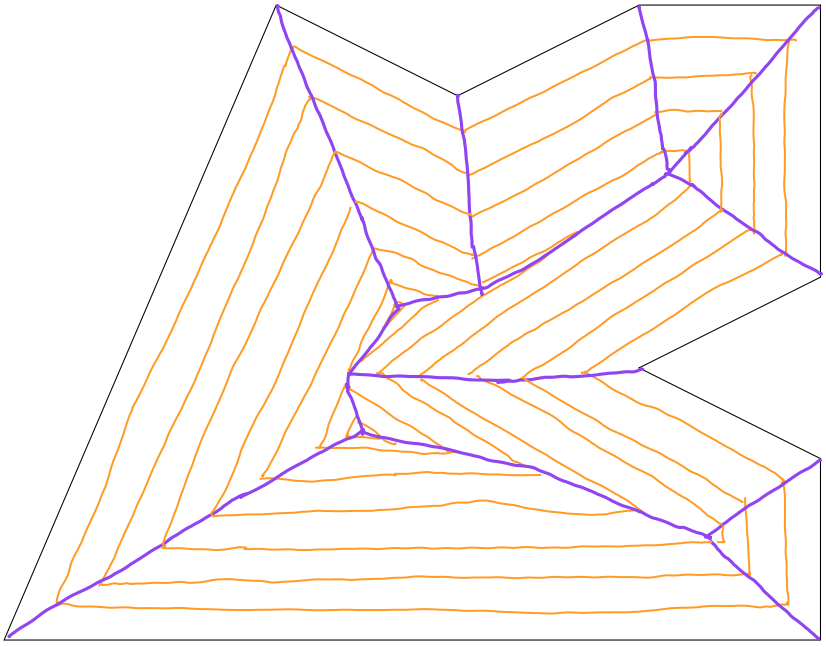
NOTE: If P is convex, then straight skeleton is the same as the medial axis.

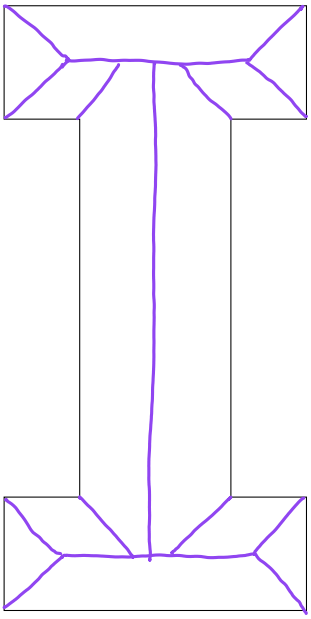
If P is not convex, then the straight skeleton avoids the curves that may occur in the medial axis.

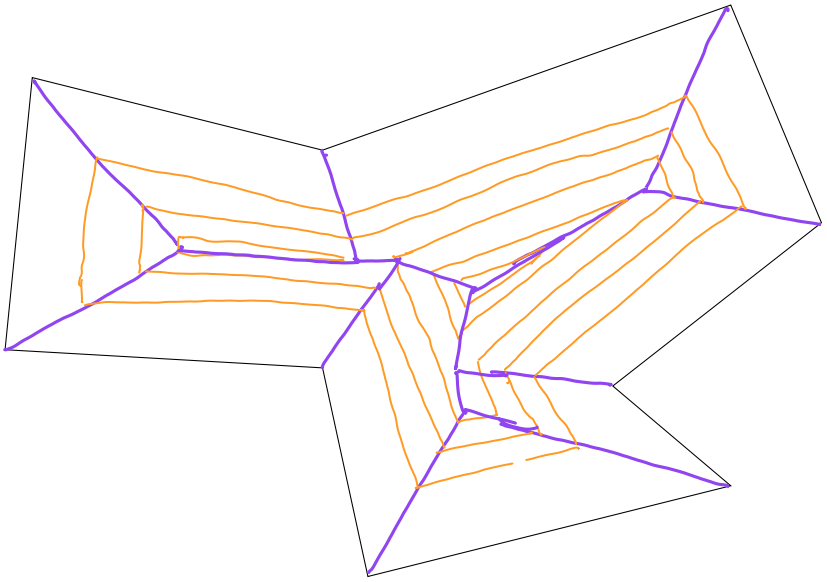
Straight Skeleton

MATH 261 Computational Geometry

1. Sketch the straight skeleton of each of the following polygons.







2. Design an algorithm to compute the straight skeleton of a polygon. What is the computational complexity of your algorithm?

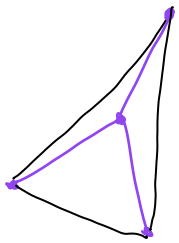
Track angle bisectors from all vertices (including reflex vertices).
Priority queue tracks two types of events:

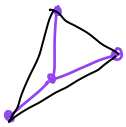
(1) Edge shrinks to zero length: two angle bisectors intersect, as in medial axis algorithm

(2) Reflex vertex collides with an edge. Then two smaller polygons are formed; continue algorithm with each.

Complexity: convex: $O(n \cdot \log(n))$ non-convex: $O(n^{17/11 + \varepsilon})$

3. Is every geometric tree the straight skeleton of some polygon? Can a geometric tree be the straight skeleton of multiple different polygons?





NO:

4. Extend the concept of straight skeleton to polygons with holes. Draw some examples. How does your algorithm adapt to this new setting?