Math 242 Challenge Problems

Spring 2023

The explorations and exercises below refer to the *Computational Mathematics* text. See the text for more details and guidance about each problem. More problems will be added to this list throughout the semester.

1. **Exploration 1.16: π as area.** Implement the algorithm outlined in this exercise (pages 23–25 in the text) to approximate π as the area of a quarter circle. Analyze and discuss the accuracy and efficiency of this method.

2. **Exploration 1.23: Create your own Machin-like formulas**. Use the methods described in Section 1.4 of the text to find your own Machin-like formulas for π. This exploration requires you to find at least three Machin-like formulas, convert each formula to a power series, implement them in Mathematica, and assess their accuracy for approximating π.

3. **Exploration 1.24: Accuracy of Machin-like formulas.** Investigate the accuracy of Machin-like formulas, such as those in Section 1.4 of the text. Formulate a precise conjecture, supported by your own computational evidence, about how the accuracy of the formula depends on the values of the a_i and b_i. (See page 29 of the text.) For this problem it is important to consider lots of Machin-like formulas and look for patterns.

4. **Exercise 1.43: Continued Fractions for π.** Read Section 1.7 of the text. Implement one of the methods for computing convergents of continued fractions. Use your code to complete Exercise 1.43.

5. **Exploration 2.21: Generalize a Fibonacci identity.** Start with the identity in the text, introduce a new index (or more than one), and conjecture a new identity.

6. **Exploration 2.22: Fibonacci identities.** Search for Fibonacci identities based on the three expressions given in the text.

7. **Exploration 2.24: Do algebraic identities lead to Fibonacci identities?** Explore possible Fibonacci identities inspired by the algebraic identities in the text.

8. **Exploration 2.34: Generalized Fibonacci polynomial identity.** Search for a general formula that gives the coefficients of the Fibonacci $(2q + 1)n$ identity for odd integers $2q + 1$.

9. **Exploration 2.37: Fibonacci identities involving sums.** Explore a class of Fibonacci identities involving $\sum_{a+b=n} F_a F_b$.

10. **Exploration 3.8: Collatz trajectory sets.** Explore the sets of integers that arise in Collatz trajectories for certain sets of starting values.

11. **Exploration 3.32: Collatz stopping times.** Explore how the maximum stopping time for Collatz trajectories grows with n.

12. **Exploration 3.33: "Horizontal segments" in Collatz stopping time plot.** Investigate the horizontal line segments that appear in the Collatz stopping time plot. How does the length of these segments increase with n?

13. **Exploration 3.38: Accelerated Collatz trajectories.** Compute accelerated Collatz trajectories for big integers. Do your computations support Estimate 3.4 in the text?

14. **Exploration 3.69: Ergodicity of logistic map trajectories.** How many iterations of the logistic map are required, on average, until the trajectory contains a point in each interval of size $\frac{1}{M}$? How does this depend on M?

15. **Exploration 4.20: a prime polynomial.** How often do polynomials produce primes? Find prime numbers produced by the polynomial $f(n)$ in the text and compare with other polynomials.

16. **Prime gaps.** A *prime gap* is the difference between successive prime numbers. The first few prime gaps are $1, 2, 2, 4, 2, 4, \ldots$. What is the distribution of prime gaps? Compute the distribution the first n prime gaps for various choices of n and report on your observations. Why are certain gaps so common? Do you think there is a most common prime gap? Explain.

17. **Exploration 4.46: Carmichael numbers.** Carmichael numbers are composite integers that are likely to be labeled as prime by primality tests based on Fermat's little theorem. Implement a function to find Carmichael numbers, as described in the text.