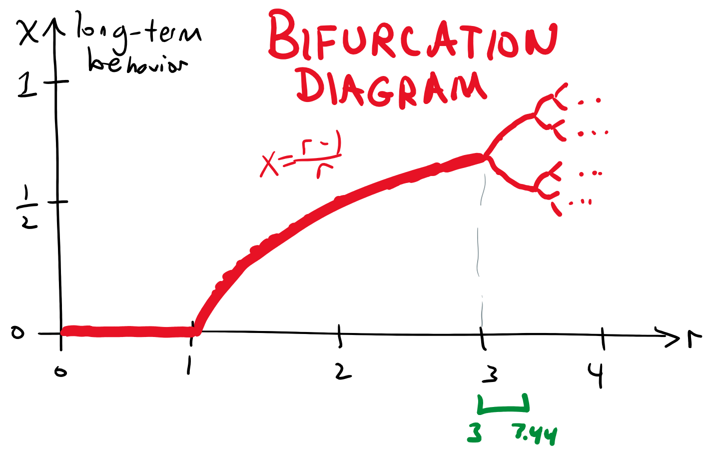
The Logistic Map

$$f_r(x) = r \cdot x(1-x), \quad \begin{array}{l} 0 \le r \le 4 \\ 0 \le x \le 1 \end{array}$$

If $0 \le r \le 1$, then trajectories converge to zero.

If $1 < r \le 3$, then trajectories converge to $x = \frac{r-1}{r}$.

If $3 < r < 3.44$, then trajectories oscillate between 2 values in the long run.



As r increases beyond 3.44, we observe a sequence of period-doubling bifurcation.

CHAOS: Chaotic trajectories satisfy:

1. They are not periodic. They never repeat exactly.
2. They visit every small interval in the domain.
3. They exhibit sensitive dependence on initial conditions.

   If you change the initial value a tiny bit, then the long-term behavior is completely different.

Investigate trajectories for $r=4$.