

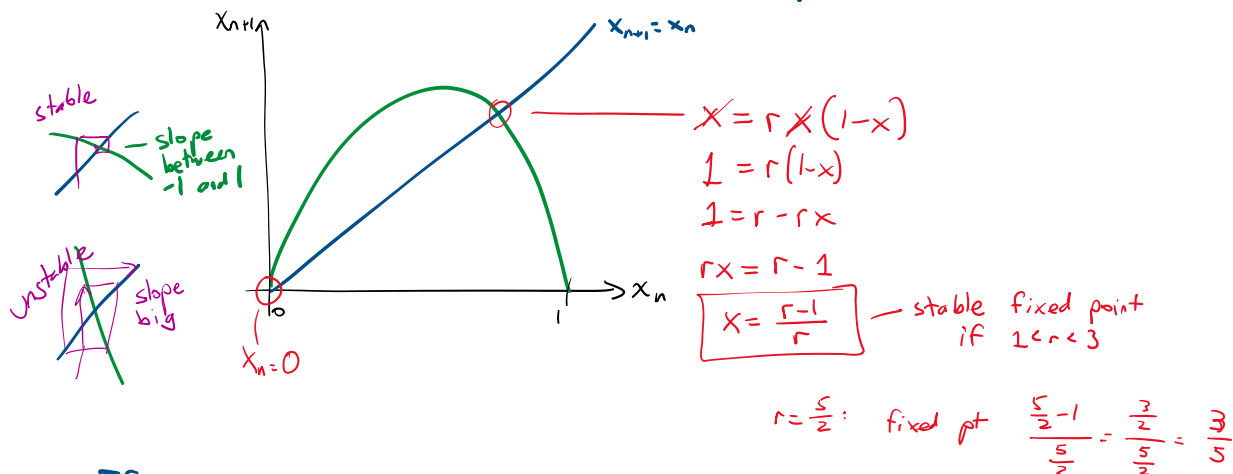
# The Logistic Map $f_r(x) = rx(1-x)$

What did you observe for  $r > 3$ ?

At what value does a phase transition occur?

How would you describe the phase transition?

If  $r < 3$ , we find a stable fixed point



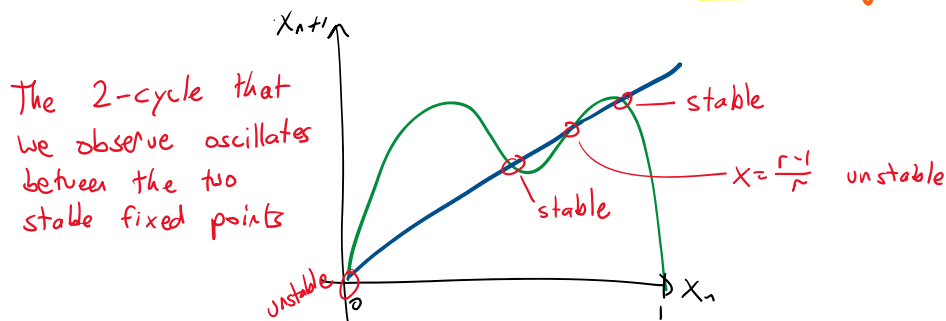
If  $r > 3$ :

Example: if  $r = 3.2$ , then  $\frac{r-1}{r} = \frac{2.2}{3.2} = \frac{22}{32} = \frac{11}{16}$

We observe alternation between 2 values, suggesting that  $f_r(f_r(x))$  has stable fixed points.

Compose  $f_r$  with itself:

$$\begin{aligned}
 f_r(f_r(x)) &= r f_r(x)(1 - f_r(x)) = r(rx(1-x))(1 - rx(1-x)) \\
 &= r^2x(1-x) - r^2x^2(1-x)^2 \quad \text{degree 4}
 \end{aligned}$$



We are observing period-doubling bifurcations:

- at  $r=3$ , a 1-cycle bifurcates into a 2-cycle
- at  $r \approx 3.45$ , 2-cycle bifurcates into a 4-cycle

change in long-term behavior

Does an 8-cycle emerge next?

How about a 16-cycle?