SIEVE OF SUNDARAM

If q is an odd composite, then $q = (2i+1)(2j+1)$ (odd, odd) for some positive integers i and j.

Then: $q = 4ij + 2i + 2j + 1 = 2(2ij + i + j) + 1$

If p is an odd prime, then $p = 2w + 1$, but $w \neq 2ij + i + j$ for any integers i and j.

ALGORITHM:

1. Start with a list $1, 2, 3, 4, \ldots, m$
2. Remove all numbers of the form $2ij + i + j$
3. For all remaining numbers, double and add 1.

EXAMPLE:

nums = 1, 2, 3, ~~4~~, 5, 6, ~~7~~, 8, 9, ~~10~~, 11, ~~12~~, ~~13~~, 14, 15

$i=1, j=1$: $2ij+i+j = 2(1)(1)+1+1 = 4$
$i=1, j=2$: $2(1)(2)+1+2 = 7$
$i=1, j=3$: $2(1)(3)+1+3 = 10$
$i=1, j=4$: $= 13$
$i=1, j=5$: $= 16$

$i=2, j=2$: $2(2)(2)+2+2 = 12$
$i=2, j=3$: $2(2)(3)+2+3 = 17$
$i=3, j=3$: $2(3)(3)+3+3 = 24$

Double and add 1:

3, 5, 7, 11, 13, 17, 19, 23, 29, 31 ← primes!

Prime Counting Function

$\pi(x)$ is the number of primes less than or equal to x

Example: $\pi(10) = 4$

Since $2, 3, 5, 7$ are the primes ≤ 10