

Sieve of Sundaram

If q is an odd composite, then $q = \underbrace{(2i+1)(2j+1)}$ for some positive integers i and j .
odd factors

$$q = 4ij + 2i + 2j + 1 = \underbrace{2(2ij + i + j)} + 1$$

If p is an odd prime, then $p = 2w + 1$, where $w \neq 2ij + i + j$ for any integers i and j .

To list odd primes:

1. Start with a list $\text{nums} = [1, 2, 3, \dots]$
2. Remove all numbers of the form $2ij + i + j$
3. For all remaining numbers: double and add 1

EXAMPLE:

nums: 1, 2, 3, ~~4~~, 5, 6, ~~7~~, 8, 9, ~~10~~, 11, ~~12~~, ~~13~~, 14, 15

$$\begin{array}{l}
 i=1, j=1: \quad i+j+2ij = 1+1+2(1)(1) = 4 \\
 i=1, j=2: \quad i+j+2ij = 1+2+2(1)(2) = 7 \\
 i=1, j=3: \quad i+j+2ij = 1+3+2(1)(3) = 10 \\
 i=1, j=4: \quad i+j+2ij = 1+4+2(1)(4) = 13 \\
 i=1, j=5: \quad \quad \quad \quad \quad \quad \quad = 16 \\
 \left. \begin{array}{l}
 i=2, j=2: \quad i+j+2ij = 2+2+2(2)(2) = 12 \\
 i=2, j=3: \quad \dots = 2+3+2(2)(3) = 17 \\
 \hline
 i=3, j=3: \quad 3+3+2(3)(3) = 24
 \end{array} \right\}
 \end{array}$$

double and add 1:

3, 5, 7, 11, 13, 17, 19, 23, 29, 31 ← ^{odd} primes

PRIME COUNTING FUNCTION

$\pi(x)$ is the number of primes less than or equal to x

Example: $\pi(10) = 4$

2, 3, 5, 7