3n identity: $F_{3n} = aF_n^3 + bF_n^2 + cF_n$

$n=2$: $8 = a+b+c$

$n=4$: $144 = 27a + 9b + 3c$

$n=6$: $2584 = 512a + 64b + 8c$

$$\to \begin{bmatrix} 8 \\ 144 \\ 2584 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 27 & 9 & 3 \\ 512 & 64 & 8 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} F_6 \\ F_{12} \\ F_{18} \end{bmatrix} = \begin{bmatrix} F_2^3 & F_2^2 & F_2 \\ F_4^3 & F_4^2 & F_4 \\ F_6^3 & F_6^2 & F_6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

5n identity: $F_{5n} = aF_n^5 + bF_n^4 + cF_n^3 + dF_n^2 + eF_n$

$n=2$
$n=4$
$n=6$
$n=8$
$n=10$

$$\begin{bmatrix} F_{10} \\ F_{20} \\ F_{30} \\ F_{40} \\ F_{50} \end{bmatrix} = \begin{bmatrix} F_2^5 & F_2^4 & F_2^3 & F_2^2 & F_2^1 \\ F_4^5 & F_4^4 & F_4^3 & F_4^2 & F_4^1 \\ \vdots & & & & \vdots \\ F_{10}^5 & & \cdots & & F_{10}^1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$$

nVals   vector   matrix

In general: Let q be an odd integer. Find the Fibonacci qn identity

nVals: Range[2, $\underline{2n}$, 2] = $\{2, 4, 6, \ldots, 2n\}$

Vector: Table[f[q·n], {n, nVals}]

matrix: Table[f[n]^i , {n, nVals}, {i, q, 1, −1}]

count down from q to 1

Verify Identity: qn-identity

$$\underbrace{F_{qn}}_{\text{lhs}} = \underbrace{a_1 F_n^q + a_2 F_n^{q-1} + a_3 F_n^{q-2} + \cdots + a_q F_n}_{\text{rhs}} = \boxed{\sum_{i=1}^{q} a_i F_n^{q-i+1}}$$

Check that lhs = rhs for n = 2, 4, 6, 8, ..., nMax