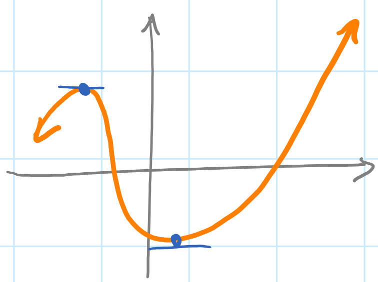
MATH 242: Wednesday, May 6

TODAY:

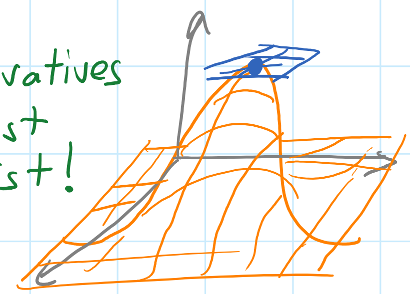
- Remarks about simulated annealing
- Finding magic squares by simulated annealing

Can't we find maxima using calculus?

YES — for continuous functions



derivatives must exist!



Our setting: <u>Combinatorial optimization</u>

We want to find an optimal object from a large, discrete search space.

<u>Examples:</u> $(n_1, n_2, \ldots, n_{10})$ $n_i \in \{0, 1, 2, 3, \ldots\}$

Magic squares:

	1	8	4	15
	6	5	3	14
	7	2	9	18
15	14	15	16	15

optimal solution:
all row, column, and diagonal sums are the same

Traveling Salesperson Problem (TSP):

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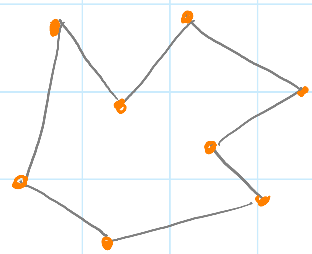
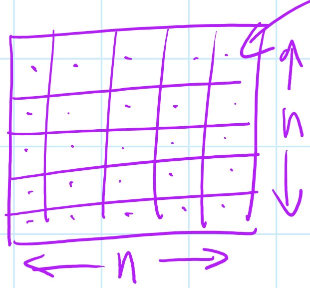


Goal: visit all locations with minimal travel distance

MAGIC SQUARES PROJECT

GOAL: Place the numbers $1, 2, 3, \ldots, n^2$ in a $n \times n$ grid such that all row, column, and diagonal sums are equal.



$1, 2, 3, \ldots, n^2$

What is the sum of

$$1 + 2 + 3 + \cdots + (n^2-1) + n^2 = \frac{n^2(n^2+1)}{2}$$

$$n^2 + (n^2-1) + (n^2-2) + \cdots + 3 + 2 + 1$$

$n^2+1 \quad n^2+1 \quad n^2+1 \quad \ldots \quad n^2+1$

n^2 column sums, each equal to n^2+1

n row sums, each equal to $\frac{1}{n}\left(\frac{n^2(n^2+1)}{2}\right) = \boxed{\frac{n(n^2+1)}{2}}$

SIMULATED ANNEALING:

- **States:** All possible arrangements of $1, 2, 3, \ldots, n^2$ in the $n \times n$ grid.
- **Function to maximize:** $m(\text{state})$ needs to indicate how far the row, col, and diagonal sums are from $\frac{n(n^2+1)}{2}$.

  minimize this!

  Maximize $-m(\text{state})$

- **Transitions between states:**

Randomly choose two entries from the grid and swap them.