Mandelbrot Set:

The set of all complex numbers c such that $f_c(z) = z^2 + c$ is bounded in absolute value when iterated from $z = 0$.

examples:

$c = 2$: $f_2(z) = z^2 + 2$ so:

$f_2(0) = 0^2 + 2 = 2$

$f_2(2) = 2^2 + 2 = 6$

$f_2(6) = 6^2 + 2 = 38$

$f_2(38) = 38^2 + 2 =$ big

diverges!

NOT in the Mandelbrot set

$c = -1$: $f_{-1}(z) = z^2 - 1$ so:

$f_{-1}(0) = 0^2 - 1 = -1$

$f_{-1}(-1) = (-1)^2 - 1 = 0$

$f_{-1}(0) = -1$ oscillation

Is in the Mandelbrot set

Mean-Median Map:

Start with a sequence $x_1, x_2, \ldots, x_n$.

Compute x_{n+1} by the following rule:

$$\frac{x_1 + x_2 + \cdots + x_n + x_{n+1}}{n+1} = \text{median}(x_1, x_2, \ldots, x_n)$$

mean (average) of $x_1, \ldots, x_n, x_{n+1}$

middle number of $x_1, \ldots, x_n$ when sorted in increasing order

Example: $n = 1$ $x_1 = 5$ compute x_2:

$$\frac{5 + x_2}{2} = \text{median}(5)$$

$$5 + x_2 = 2 \cdot 5$$

$$x_2 = 5$$

$$5 + x_2 = 2 \cdot 5$$
$$x_2 = 5$$

compute: x_3: $\dfrac{5+5+x_3}{3} = \text{median}(5,5)$

$$10 + x_3 = 3 \cdot 5$$
$$x_3 = 5$$

Similarly, $x_4 = 5$, $x_5 = 5$, ...

Example: $n=2$ $x_1 = 7$, $x_2 = 2$

compute x_3: $\dfrac{7+2+x_3}{3} = \text{median}(7,2)$

$$7+2+x_3 = 3 \cdot \frac{9}{2}$$
$$x_3 = \frac{27}{2} - 9 = \frac{9}{2}$$

Compute x_4: $\dfrac{7+2+\frac{9}{2}+x_4}{4} = \text{median}\left(7, 2, \frac{9}{2}\right)$

$$7+2+\frac{9}{2}+x_4 = 4 \cdot \frac{9}{2}$$
$$\frac{27}{2} + x_4 = \frac{36}{2}$$
$$x_4 = \frac{36-27}{2} = \frac{9}{2}$$

Compute $x_5 = \frac{9}{2}$, $x_6 = \frac{9}{2}$, ...

Things are more interesting if $n=3$.

Note: $x_{n+1} = (n+1)\,\text{median}(x_1, \dots, x_n) - (x_1 + x_2 + \cdots + x_n)$