

Last time:

Pell numbers

$$P_{3n} = 8 P_n^3 + (-1)^n 3 P_n$$

$$P_{5n} = 64 P_n^5 + (-1)^n 40 P_n^3 + 5 P_n$$

$$P_{7n} = a P_n^7 + b P_n^5 + c P_n^3 + d P_n \quad \text{for even } n$$

where should $(-1)^n$ go?

Collatz Function:

$$c(n) = \begin{cases} 3n+1 & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Let $n=5$ and iterate $c(n)$: $c(5) = 3(5)+1 = 16$, $c(16) = \frac{16}{2} = 8$,

$$c(8) = \frac{8}{2} = 4, \quad c(4) = \frac{4}{2} = 2, \quad c(2) = \frac{2}{2} = 1$$

$$c(1) = 3(1)+1 = 4, \quad c(4) = 2, \quad \dots \text{ repeat } \dots$$

Start with 33:

$$\begin{aligned} 33 &\rightarrow 100 \rightarrow 50 \rightarrow 25 \rightarrow 76 \rightarrow 38 \rightarrow 19 \\ &\rightarrow 58 \rightarrow 29 \rightarrow 88 \rightarrow 44 \rightarrow 22 \rightarrow 11 \rightarrow 34 \\ &\rightarrow 17 \rightarrow 52 \rightarrow 26 \rightarrow 13 \rightarrow 40 \rightarrow 20 \rightarrow 10 \\ &\rightarrow 5 \rightarrow 16 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow \dots \end{aligned}$$

Start with 7:

$$7 \rightarrow 22 \rightarrow 11 \rightarrow 34 \rightarrow \dots$$

Collatz Conjecture: Starting with any positive integer n , the sequence of Collatz iterates eventually reaches 1.

This has been verified for n up to about 10^{20} .