

RANDOM WALKS

1D: average number of returns to origin: $\sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi n}}$ DIVERGES

Random walk returns infinitely often to the origin.

2D: average number of returns to origin: $\sum_{n=1}^{\infty} \frac{1}{\pi n}$ DIVERGES

Random walk returns infinitely often to the origin.

But: time between returns might be very big!

3D: average number of returns to origin: $\sum_{n=1}^{\infty} \frac{1}{(\pi n)^{3/2}}$ CONVERGES!

Random walk returns only finitely many times to the origin. $\frac{3}{2} > 1$

For a simple 3D random walk, the probability of returning to the origin is about 34%.

PERCOLATION THEORY

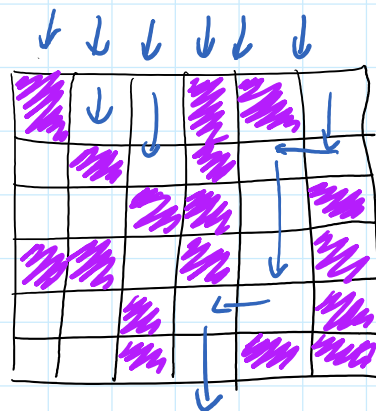
Start with a grid of squares.

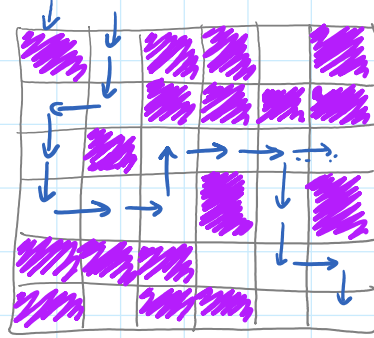
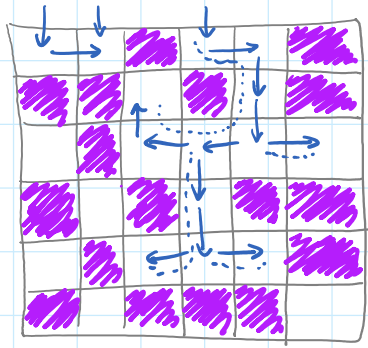
Fix a probability p .

For each square, color it purple with probability p .

Regard the white squares as "open" and the colored squares as "closed".

Pour water on top of the grid. Can it flow through the open squares to the bottom of the grid?





ALGORITHM: Depth-First Search

How to implement this in code?