SIEVE OF ERATOSTHENES

$\boxed{2}, \boxed{3}, \cancel{4}, \boxed{5}, \cancel{6}, 7, \cancel{8}, \cancel{9}, \cancel{10}, 11, \cancel{12}, 13, \cancel{14}, \cancel{15}, \cancel{16}, 17, \cancel{18}, 19, \cancel{20}$

$2, 3, \cancel{4}, 5, \cancel{6}, 7, \cancel{8}, \cancel{9}, \cancel{10}, 11, \cancel{12}, 13, \cancel{14}, \cancel{15}, \cancel{16}, 17, \cancel{18}, 19, \cancel{20}$

boolvals[[i^2 ;; n ;; i]] = False

$i=2$: set positions 4, 6, 8, 10, ... to False

$i=3$: set positions 9, 12, 15, 18, ... to False

$i=4$: boolVals[[4]] is already False, so do nothing

$i=5$: set positions 25, 30, 35, ... to False

SIEVE OF SUNDARAM

Algorithm:

1. Start with a positive integer n.

2. List 1 = list of all integers of the form $i+j+2ij$, where i and j are integers, $1 \le i \le j$, and $i+j+2ij \le n$.

3. List 2 = list of all integers in $1, 2, 3, \ldots, n$ that are <u>not</u> in List 1.

4. For each number in List 2, double it and add 1.
   This gives all of the odd primes up to $2n+1$.

Why does this work?

The numbers resulting from the algorithm are odd integers q, with $3 \le q \le 2n+1$.

The numbers not in List2 are precisely the composite odd integers.

We removed numbers of the form:

$$2(i+j+2ij)+1 \qquad \text{for integers } i,j$$

$$= \underbrace{4ij + 2i}_{2i(2j+1)} + \underbrace{2j + 1}_{+1(2j+1)}$$

$$= (2i+1)(2j+1)$$ ← two odd factors, which are at least 3