How many primes are there? Infinitely many.

PROOF: Assume there are finitely many primes

$$2, 3, 5, 7, \ldots, p$$

Then let $n = (2 \cdot 3 \cdot 5 \cdot 7 \cdot \cdots \cdot p) + 1$.

Note: n is not divisible by $2, 3, \ldots, p$ (not divis. by any prime)

So n is prime, which gives a contradiction.

Thus, There are infinitely many primes.

CURRENT RECORDS:

- Biggest known prime: $2^{82,589,933} - 1$ (24 million digits)
- Arbitrary numbers up to about 15,000 digits can be proven prime.
- Arbitrary numbers up to about 200 digits can be factored.
- 2018: 230-digit number (RSA-230) was factored into 2 prime factors

PRIMALITY TEST VIA TRIAL DIVISION

given integer $n \geq 2$:

Loop[$d = 2$ to $\sqrt{n}$:
    If[d divides n, then return False.]]

return True

Given integer $n \geq 2$.

found Divisor = False

loop[$d = 2$ to $\sqrt{n}$:
    If[d divides n, then found Divisor = True]]

return ! found Divisor

A Mathematica implementation:

```
myPrimeQ[n_] := Module[
  {foundDivisor = False},

  Do[
   If[Divisible[n, k], foundDivisor = True ]
   , {k, 2, Sqrt[n]}];

  ! foundDivisor
 ]
```