

# MEAN - MEDIAN MAP

Start with a sequence  $x_1, x_2, \dots, x_n$ .

Compute  $x_{n+1}$  by the following rule:

$$\underbrace{\frac{x_1 + x_2 + \dots + x_n + x_{n+1}}{n+1}}_{\text{mean of } x_1, \dots, x_{n+1}} = \text{median}(x_1, \dots, x_n)$$

Repeat.

Example:

$$n=1$$

$$x_1 = 5$$

Then:  $\frac{5 + x_2}{2} = \text{median}(5) = 5$

$$x_2 = 5$$

$$\frac{5 + 5 + x_3}{3} = \text{median}(5, 5) = 5$$

$$x_3 = 5 \quad x_3 = x_4 = x_5 = \dots$$

$$n=2$$

$$x_1 = 7$$

$$x_2 = 2$$

Then:  $\frac{7 + 2 + x_3}{3} = \text{median}(7, 2) = \frac{9}{2}$

$$x_3 = 3\left(\frac{9}{2}\right) - 7 - 2 = \frac{9}{2} = x_3$$

$$\frac{7 + 2 + \frac{9}{2} + x_4}{4} = \text{median}\left(7, 2, \frac{9}{2}\right) = \frac{9}{2}$$

$$x_4 = 4\left(\frac{9}{2}\right) - 7 - 2 - \frac{9}{2} = \frac{9}{2} = x_4$$

$$x_5 = \frac{9}{2} = x_6 = \dots$$

$$n=3$$

$$x_1 = 4, \quad x_2 = 5, \quad x_3 = 8$$

$$\frac{4 + 5 + 8 + x_4}{4} = \text{median}(4, 5, 8) = 5$$

$$x_4 = 4(5) - 4 - 5 - 8 = 3$$