THEOREM: Let I be an interval and $f: I \to I$.

If f has a point of period 3, then f has a point of period k for every positive integer k.

proof by Li and Yorke (1975)

follows from Sharkovsky's Theorem (1964)

COLLATZ FUNCTION:
$$c(n) = \begin{cases} 3n+1 & \text{if } n \text{ odd} \\ \frac{n}{2} & \text{if } n \text{ even} \end{cases}$$

generalization:
$$f(z) = [1+\cos(\pi z)]\frac{z}{4} + [1-\cos(\pi z)]\frac{3z+1}{2}$$

if n odd: $\cos(\pi n) = -1$ so
$$f(n) = [1-1]\frac{z}{4} + [1+1]\frac{3n+1}{2} = 3n+1$$

if n even: $\cos(\pi n) = 1$ so
$$f(n) = [1+1]\frac{n}{4} + 0 = \frac{n}{2}$$

COMPLEX PLANE: $z = x + iy$ $\quad x, y \in \mathbb{R}$, $\quad i = \sqrt{-1}$

$i^2 = 1$