Cassini's Identity

$$F_{n-1}F_{n+1} - F_n^2 = (-1)^n$$

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$ for $n > 1$

Proof: Start with the matrix formula:

$$\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n$$

← This can be derived by mathematical induction

$$\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} F_{n+1}+F_n & F_{n+1} \\ F_n + F_{n-1} & F_n \end{bmatrix} = \begin{bmatrix} F_{n+2} & F_{n+1} \\ F_{n+1} & F_n \end{bmatrix}$$

Take determinants.

$$\det\begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix} = \det\left(\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n\right)$$

$$F_{n+1}F_{n-1} - F_n^2 = \left(\det\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)^n$$

$$\boxed{F_{n+1}F_{n-1} - F_n^2 = (-1)^n}$$