Math 234

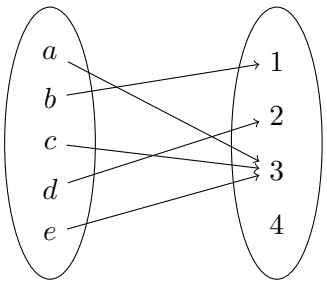
Functions Day 18

1. A student defines a function $g : \mathbf{Q} \to \mathbf{Z}$ by the rule

$$g\left(\frac{m}{n}\right) = m - n$$

for all rational numbers $\frac{m}{n}$. Is this function well defined? Why or why not?

2. Let $X = \{a, b, c, d, e\}$ and $Y = \{1, 2, 3, 4\}$. Define the function $f : X \to Y$ by the following arrow diagram.



(a) What are the values of $f(a)$, $f(b)$ and $f(c)$?

(b) What is the *domain* of f?

(c) What is the *codomain* of f?

(d) What is the *range* of f?

(e) Is a an *inverse image* of 3?

(f) What is the *inverse image* of 3?

(g) Is f well defined?

(h) Is f one-to-one?

(i) Is f onto?

3. Let $F : \mathscr{P}(\mathbf{Z}) \to \mathbf{R}$ be the function defined by

$$F(A) = \max(A) - \min(A).$$

For example, $F(\{-2, 5, 2\}) = 5 - (-2) = 7$.

(a) What is the value of $F(\{1, 3, 19, -3\})$?

(b) What is the *domain* of F?

(c) What is the *codomain* of F?

(d) What is the *range* of F?

(e) Is $\{5, -7, 2, 15\}$ an *inverse image* of 20? ...of 22?

(f) What is the *inverse image* of 6?

(g) Is F well defined?

(h) Is F one-to-one?

(i) Is F onto?

4. Draw your own arrow diagram to define a function that is one-to-one but not onto. Then draw your own arrow diagram to define a function that is onto but not one-to-one.

5. Define $f : \mathbf{R} \to \mathbf{R}$ by the rule $f(x) = 5x + 3$.

   (a) Is f one-to-one? Prove that your answer is correct.

   (b) Is f onto? Prove that your answer is correct.

6. Define $g : \mathbf{Z} \to \mathbf{Z}$ by the rule $g(n) = 3n + 1 \pmod 7$.

   (a) Is g one-to-one? Prove that your answer is correct.

   (b) Is g onto? Prove that your answer is correct.

The **floor function** assigns to each $x \in \mathbf{R}$ the largest integer that is less than or equal to x. The value of the floor function at x is denoted $\lfloor x \rfloor$.

The **ceiling function** assigns to each $x \in \mathbf{R}$ the smallest integer that is greater than or equal to x. The value of the ceiling function at x is denoted $\lceil x \rceil$.

7. Compute the following:

$$\left\lfloor \frac{1}{2} \right\rfloor = \qquad \left\lceil \frac{5}{2} \right\rceil = \qquad \lfloor 3.2 \rfloor = \qquad \lceil 7 \rceil = \qquad \lfloor -3 \rfloor =$$

8. Prove or disprove the following statements about the floor and ceiling functions.

   (a) $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$ for all real numbers x.

   (b) $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers x and y.

   (c) $\left\lceil \frac{\lceil x/2 \rceil}{2} \right\rceil = \lceil \frac{x}{4} \rceil$ for all real numbers x.

   (d) $\left\lfloor \sqrt{\lceil x \rceil} \right\rfloor = \lfloor \sqrt{x} \rfloor$ for all positive real numbers x.