

# Math 234

## Logical Statements

Day 3

Discuss the following problems with the people at your table.

1. **Warm-Up:** Write the converse, inverse, and contrapositive of the statement:

If it is hot outside, then I will buy ice cream.

← Same truth value

(a) Converse:

If ice cream, then hot outside.

same truth value

(b) Inverse:

If not hot, then not buy ice cream.

(c) Contrapositive:

If not buy ice cream, then not hot outside.

2. Complete the following truth table.

$p$	$q$	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$p$  if and only if  $q$   
 $p \oplus q$  exclusive or

$\leftrightarrow$  denotes biconditional

How does  $p \leftrightarrow q$  relate to  $p \rightarrow q$  and  $q \rightarrow p$ ?

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$p \leftrightarrow q \equiv \sim(p \oplus q) \equiv p \oplus \sim q$$

exclusive nor

3. What is the negation of the biconditional statement  $p \leftrightarrow q$ ? (Consider extending the truth table above.)

$$\sim(p \leftrightarrow q) \equiv p \oplus q$$

↘ exclusive or

4. Express the following as conditional statements.

(a)  $p$  is a necessary condition for  $q$

$$\sim p \rightarrow \sim q \quad \text{If not } p, \text{ then not } q. \quad \text{If } q, \text{ then } p. \quad q \rightarrow p$$

(b)  $r$  is a sufficient condition for  $s$

$$\text{If } r, \text{ then } s. \quad r \rightarrow s \quad \text{If not } s, \text{ then not } r. \quad \sim s \rightarrow \sim r$$

(c)  $t$  is a necessary and sufficient condition for  $u$

$$t \leftrightarrow u \quad t \text{ if and only if } u.$$

$$u \leftrightarrow t \quad u \text{ if and only if } t.$$

5. Come up with statements  $p$  and  $q$  such that...

(a) ... $p$  is a sufficient condition for  $q$ .

(b) ... $p$  is a necessary condition for  $q$ .

(c) ... $p$  is a necessary and sufficient condition for  $q$ .

(d) ... $p$  is neither a necessary nor sufficient condition for  $q$ .

6. Use symbols to write the logical form of each argument below. Then use a truth table to determine the validity of the argument. In your truth table, indicate which columns correspond to the premises and which correspond to the conclusion.

Argument 1:

$p$ 
 $q$

If Lauren spells the word incorrectly, she loses the spelling bee.
Lauren spells the word correctly.

$\therefore$  Lauren wins the spelling bee.

$p \rightarrow q$   
 $\sim p$   
 $\therefore \sim q$

$\left. \begin{array}{l} p \rightarrow q \\ \sim p \end{array} \right\} \text{premises}$   
 $\left. \begin{array}{l} \sim q \end{array} \right\} \text{conclusion}$

$p$	$q$	$p \rightarrow q$	$\sim p$	$\sim q$
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

both premises true  $\rightarrow$

since premises are true but the conclusion is false, the argument is not valid

(problem 6, continued)

Argument 2:

A rock contains either gold or pyrite (fool's gold).  
If the rock contains gold, it does not contain pyrite.  
The rock contains pyrite.  
∴ The rock does not contain gold.

p: rock contains gold  
q: rock contains pyrite

$p \vee q$   
 $p \rightarrow \sim q$   
 $q$   
∴  $\sim p$

} premises  
} conclusion

p	q	$\sim p$	$\sim q$	$p \vee q$	$p \rightarrow \sim q$
T	T	F	F	T	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	F	T

Handwritten annotations: Arrows labeled "premises" point to the columns for q, ~p, p ∨ q, and p → ~q. A yellow oval highlights the row (F, T, T, F, T, T). A purple oval highlights the column for ~p. A green circle highlights the 'T' in the ~p column of the highlighted row. A green arrow points from the text "argument is valid!" to the circled 'T'. The word "Conclusion" is written below the ~p column.

7. Solve the following logic puzzle to find the treasure, assuming that each statement below is true.

📖 this is problem 37 on page 62 of the text

- If this house is next to a lake, then the treasure is not in the kitchen.
- If the tree in the front yard is an elm, then the treasure is in the kitchen.
- This house is next to a lake.
- The tree in the front yard is an elm or the treasure is buried under the flagpole.
- If the tree in the back yard is an oak, then the treasure is in the garage.

Did you use modus ponens or modus tollens in your reasoning above?

8. A set of premises and a conclusion are given below. Show how the conclusion follows logically from the premises.

premises  $\left\{ \begin{array}{l} p \vee q \\ q \rightarrow r \\ p \wedge s \rightarrow t \\ \sim r \\ \sim q \rightarrow u \wedge s \end{array} \right.$   
 conclusion  $\rightarrow \therefore t$

(1)  $q \rightarrow r$   
 $\sim r$   
 $\therefore \sim q$   
 contrapositive  
 Modus tollens

(2)  $p \vee q$   
 $\sim q$   
 $\therefore p$   
 elimination

(3)  $\sim q \rightarrow u \wedge s$   
 $\sim q$   
 $\therefore u \wedge s$   
 implication  
 Modus ponens

(4)  $u \wedge s$   
 $\therefore s$   
 specialization

(5)  $p$   
 $s$   
 $\therefore p \wedge s$   
 conjunction

(6)  $p \wedge s \rightarrow t$   
 $p \wedge s$   
 $\therefore t$   
 Modus ponens

**BONUS:** Identify each valid argument form (as listed in Table 2.3.1 on page 60 of the text) used in your reasoning above.