

Families of Linear Systems Lab

Math 230

due Tuesday, April 19 at 4pm

In this lab you will investigate four different two-parameter families of linear systems of differential equations. In each case the parameters are two real numbers, a and b . Your goal is to provide a picture of the “parameter plane” (i.e., the a, b -plane) for each family of linear systems. *Note:* We are not talking about the trace-determinant plane here; the horizontal axis should be the a -axis, and the vertical axis should be the b -axis.

For each family, first write the matrix representation of the system and find all eigenvalues (which, of course, depend on both a and b). Then determine the sets of points (a, b) for which there are repeated eigenvalues, zero eigenvalues, or purely imaginary eigenvalues, and draw (i.e. graph) these sets in the a, b -plane.

Next, in each region between the curves that you graphed, determine the type of phase plane that occurs (e.g., saddle, spiral sink, etc.). Also indicate what points (a, b) result in repeated zero eigenvalues.

You need not display the direction that solutions travel around the origin when the eigenvalues are complex, and you do not need to compute or display eigenvectors.

Finally, for each family, write a paragraph addressing the following: As the values of a and b change so that the point (a, b) moves from one region to another, the type of linear system changes—that is, a bifurcation occurs. Which of these bifurcations is important for the long-term behavior of solutions? Which of these bifurcations corresponds to a dramatic change in the phase plane or the $x(t)$ - and $y(t)$ -graphs?

Here are the four families:

Family 1:

$$\begin{aligned}\frac{dx}{dt} &= ay \\ \frac{dy}{dt} &= bx\end{aligned}$$

Family 2:

$$\begin{aligned}\frac{dx}{dt} &= ax - by \\ \frac{dy}{dt} &= \frac{b}{4}x\end{aligned}$$

Family 3:

$$\begin{aligned}\frac{dx}{dt} &= 2bx + (a + 1)y \\ \frac{dy}{dt} &= (a - 1)x\end{aligned}$$

Family 4:

$$\begin{aligned}\frac{dx}{dt} &= 2bx - y \\ \frac{dy}{dt} &= (\sin^2 a)x\end{aligned}$$

It is recommended that you type your answers, though drawing diagrams by hand is completely acceptable. You may turn in your work either on paper or electronically on Moodle.