Euler's Method Lab

Math 230

due Thursday, February 25 at 4pm

We now have some experience using Euler's method to approxiate the solutions of differential equations. In this lab, we will investigate how the step size affects the accuracy of Euler's method.

You may (and should) use technology to help you answer the questions below, but be sure to provide the requested information. Please type your answers.

Part I

1. Let A be the day of the month on which you were born. For example, if you were born on February 16, then $A = 16$.

   Consider the initial-value problem

   $$\frac{dy}{dt} = -2t + 3, \qquad y(0) = A.$$

   Find the exact solution $y(t)$ to this problem and determine $y(1)$. Be sure to check that your answer here is correct and show this computation explicitly.

2. Use Euler's method with a step size of $\Delta t = 0.1$ to approximate $y(1)$. That is, compute in succession

   $$t_0, y_0, t_1, y_1, \ldots, t_{10}, y_{10}$$

   where $t_0 = 0$, $y_0 = A$ and $t_{10} = 1$, so that y_{10} is an approximation of $y(1)$. List in table form the values you find for t_0, $y_0, \ldots, y_{10}$. Highlight your approximate value for $y(1)$ using this step size. What is the error (the difference between your approximate value and the actual value of $y(1)$)?

3. Repeat item 2 with step size of $\Delta t = 0.05$ and twice as many steps.

4. Repeat item 2 with step size of $\Delta t = 0.01$ and ten times as many steps. You don't have to list all of the data here; just give the approximation to $y(1)$ that you find using this step size and the error.

5. In a brief essay (no more than a few paragraphs), discuss the improvement of the accuracy of Euler's method as you make the step size smaller by a factor of $\frac{1}{2}$ and $\frac{1}{10}$. How does this affect your approximation of $y(1)$? By what percent does the error decrease?

Part II

1. Let B be the number of the month in which you were born. For example, if you were born February, then $B = 2$.

   Consider the initial-value problem

   $$\frac{dy}{dt} = 2y + 3, \qquad y(0) = \frac{B}{10}.$$

   Find the exact solution $y(t)$ to this problem and determine $y(1)$. Be sure to check that your answer here is correct and show this computation explicitly.

2. Repeat items 2 – 5 above, approximating $y(1)$ using Euler's method with different step sizes. Then discuss your findings in a brief essay.

based on a lab by Robert L. Devaney: `http://math.bu.edu/people/bob/MA226/euler-lab-s16.html`