

Hamiltonian Systems

Math 230

1. Consider the system

$$\begin{aligned}\frac{dx}{dt} &= y \\ \frac{dy}{dt} &= x^2 - a\end{aligned}$$

where a is a parameter.

(a) Verify that this system is Hamiltonian. What is the Hamiltonian function?

(b) What are the equilibrium points of the system? Are there any bifurcations?

(c) Linearize the system at each of the equilibrium points and determine the behavior of solutions near the equilibrium points.

(d) Describe the bifurcation that takes place at $a = 0$. That is, how do the solutions change as a changes from negative to positive?

2. Consider the *linear* system:

$$\frac{d\mathbf{Y}}{dt} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mathbf{Y}$$

(a) What conditions on a, b, c, d guarantee that the system is Hamiltonian?

(b) If the system is Hamiltonian, then what is the Hamiltonian function?

(c) If the system is Hamiltonian, then what is the type of equilibrium point at $(0, 0)$?