TOOLS FOR QUALITATIVE ANALYSIS OF NONLINEAR SYSTEMS

1. LINEARIZATION: Use the Jacobian matrix to approximate a nonlinear system near an equilibrium point by a linear system.

2. NULLCLINES: Identify where solution curves have horizontal or vertical tangents, and infer long-term behavior of solutions.

3. UNIQUENESS THEOREM: For an autonomous system of diff. eq.'s defined by functions with continuous partial derivatives, solution curves do not intersect.

$$\begin{cases} \frac{dx}{dt} = f(x,y) \\ \frac{dy}{dt} = g(x,y) \end{cases}$$

$\begin{pmatrix} \text{no } t \text{ on} \\ \text{the right} \\ \text{side of} \\ \text{the eq.'s} \end{pmatrix}$

$$\begin{cases} \dfrac{dx}{dt} = x(1-x-ay) \\[2mm] \dfrac{dy}{dt} = 2y\left(1-\dfrac{x}{2}-y\right) \end{cases}$$

1. $a = \dfrac{1}{2}$:

$$\begin{cases} \frac{dx}{dt} = x\left(1-x-\frac{1}{2}y\right) \\[2mm] \frac{dy}{dt} = 2y\left(1-\frac{1}{2}x-y\right) \end{cases}$$

x-nullclines: $x=0$, $y=2-2x$

y-nullclines: $y=0$, $y=1-\frac{x}{2}$

saddle

sink

source

eigenvectors
$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

saddle
$\lambda = -1,\ \lambda = 1$
$\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\vec{v}_2 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$