Linear Systems with Complex Eigenvalues

Math 230

Consider the matrix $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

1. Find the eigenvalues of $\mathbf{A}$. Note that they are complex conjugates.

2. Let λ be the eigenvalue with positive imaginary part. Find an eigenvector $\mathbf{V}$ associated with λ.

3. Write down the complex-valued "straight-line" solution $\mathbf{Y}_1 = \mathbf{V}e^{\lambda t}$ to the system $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$.

4. Use Euler's formula, $e^{i\theta} = \cos\theta + i\sin\theta$, to expand this solution. Then collect the real and imaginary parts so that $\mathbf{Y}_1 = \mathbf{Y}_{\text{re}} + i\mathbf{Y}_{\text{im}}$, where $\mathbf{Y}_{\text{re}}$ and $\mathbf{Y}_{\text{im}}$ are real-valued functions.

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5. Show that $\mathbf{Y}_{\text{re}}$ and $\mathbf{Y}_{\text{im}}$ each satisfy the system $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$.

6. Write down the general solution to $\frac{d\mathbf{Y}}{dt} = \mathbf{AY}$.

7. Find a solution with the initial value $\mathbf{Y}(0) = (2, 4)$.

8. What is the long-term behavior of the solution you found?