EXAM 1:

A: 68 – 76
B: 59 – 67
C: 50 – 58
D: less than 50

rough guide for converting numerical scores to letter grades

TODAY: Continue studying $\frac{d\vec{Y}}{dt} = A\vec{Y}$, where A is a 2×2 matrix

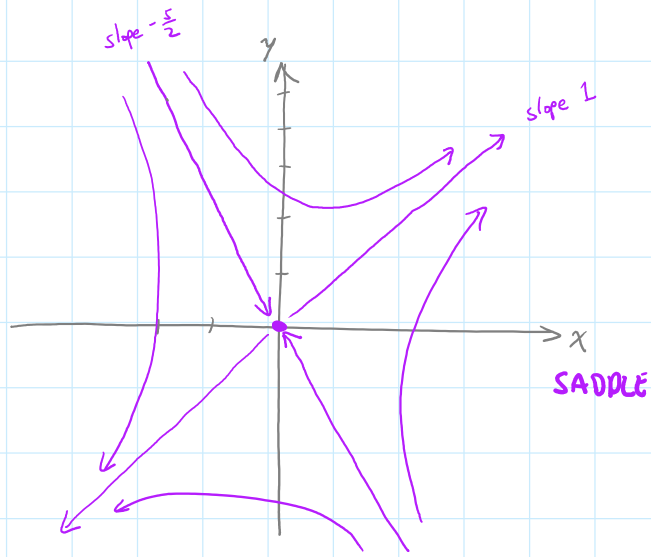
1. $A = \begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$

eigenvalues: $\lambda_1 = -3, \ \lambda_2 = 4$

eigenvectors: $\vec{v}_1 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

general solution:

$$\vec{Y}(t) = k_1 e^{-3t} \begin{bmatrix} -2 \\ 5 \end{bmatrix} + k_2 e^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



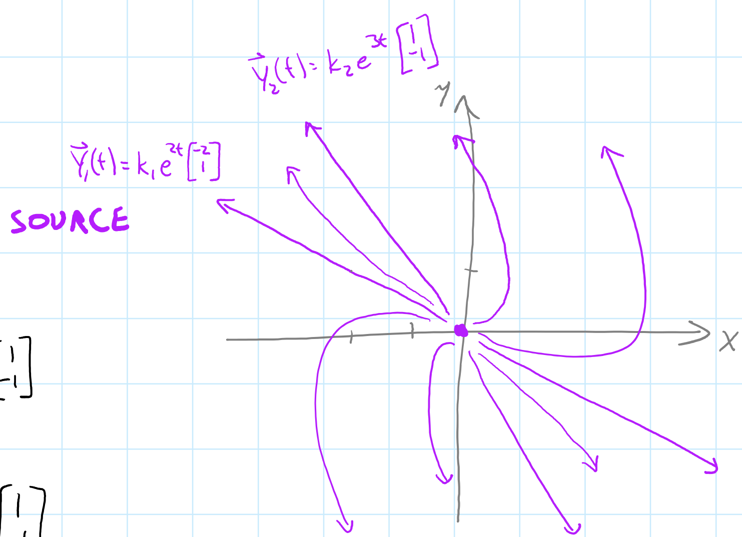
2. $A = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$

eigenvalues: $\lambda_1 = 2, \ \lambda_2 = 3$

eigenvectors: $\vec{v}_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

general solution:

$$\vec{Y}(t) = k_1 e^{2t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + k_2 e^{3t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



3. $A = \begin{bmatrix} -2 & 2 \\ 2 & -5 \end{bmatrix}$

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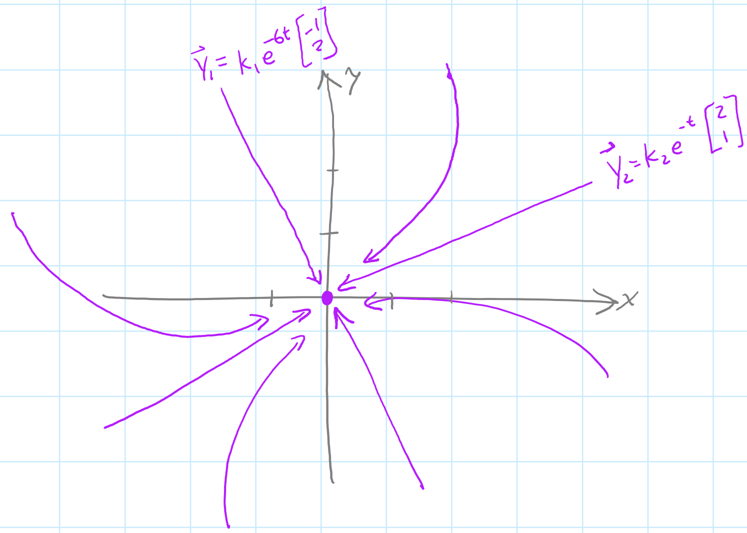

eigenvalues: $\lambda_1 = -6, \quad \lambda_2 = -1$

eigenvectors: $\vec{v}_1 = \begin{bmatrix}-1\\2\end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix}2\\1\end{bmatrix}$

general solution:

$$\vec{Y}(t) = \underbrace{k_1 e^{-6t}\begin{bmatrix}-1\\2\end{bmatrix}}_{\text{goes to zero faster}} + \underbrace{k_2 e^{-t}\begin{bmatrix}2\\1\end{bmatrix}}_{\text{goes to zero slower}}$$

NOTE: We can now solve $\frac{d\vec{Y}}{dt} = A\vec{Y}$ when A has two distinct real eigenvalues $\lambda_1 \neq \lambda_2$.

Consider: $A = \begin{bmatrix} 1 & 5 \\ -1 & -3 \end{bmatrix}$ $\qquad \frac{d\vec{Y}}{dt} = A\vec{Y}$

$$\det(A - \lambda I) = \det\begin{bmatrix} 1-\lambda & 5 \\ -1 & -3-\lambda \end{bmatrix} = (1-\lambda)(-3-\lambda) + 5 = \lambda^2 + 2\lambda - 3 + 5$$

We want: $\lambda^2 + 2\lambda + 2 = 0$

$i = \sqrt{-1}$

So: $$\lambda = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

Eigenvalues: $\lambda_1 = -1 + i, \quad \lambda_2 = -1 - i$ $\leftarrow$ conjugate pair of complex eigenvalues

Eigenvectors: to be determined