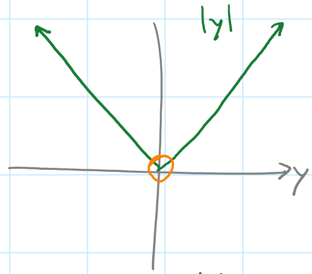
EXISTENCE AND UNIQUENESS

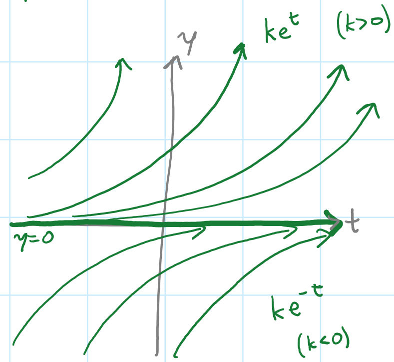


#2 from last time: $\frac{dy}{dt} = |y|$ $f(t,y) = |y|$

$f(t,y) = |y|$ is continuous everywhere, so there exists a solution to the diff. eq. through any point (t,y)

$f(t,y) = |y|$ is differentiable, except at $y=0$, so the solution is unique through (t,y), $y \neq 0$

Find solutions: $\frac{dy}{dt} = |y|$



- If $y>0$: $\frac{dy}{dt} = y$ $y(t) = ke^t$, $k>0$.

  deriv. equals function

- If $y<0$: $\frac{dy}{dt} = -y$ $y(t) = ke^{-t}$, $k<0$.

  deriv. equals neg. itself

- $y=0$ is an equilibrium solution

  $\frac{dy}{dt} = 0$

We see that solutions through $(t,0)$ are unique, but the theorem didn't tell us that.

EXAMPLE: from video on existence & uniqueness

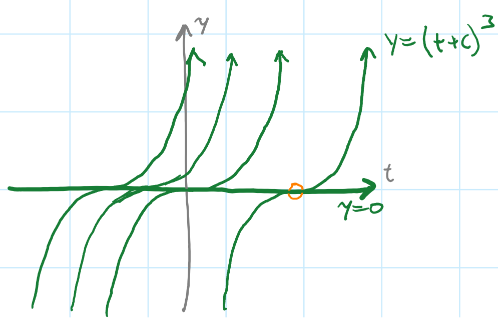
$$\frac{dy}{dt} = \underbrace{3y^{2/3}}_{f(t,y)}$$

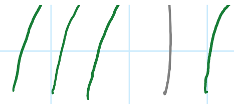
$$\frac{\partial f}{\partial y} = 3 \cdot \frac{2}{3} y^{-\frac{1}{3}} = \frac{2}{y^{1/3}}$$

$\frac{\partial f}{\partial y}$ not defined at $y=0$, so the theorem doesn't tell us whether solutions are unique at $(t,0)$.

Solutions: $y=0$

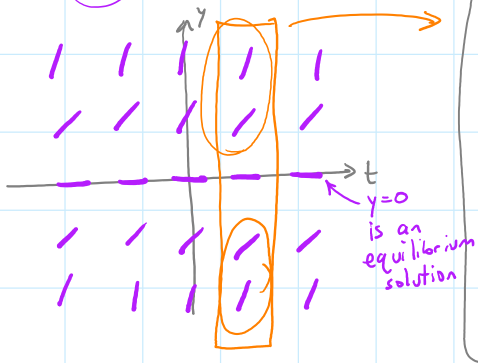
$y = (t+C)^3$



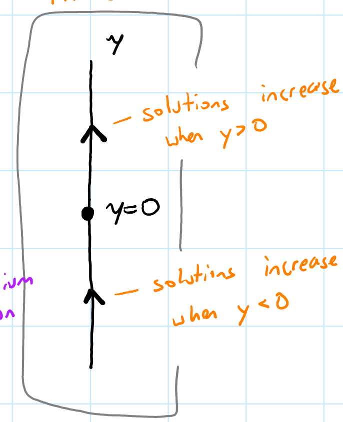


PHASE LINE: a "vertical slice" of the slope field for $\frac{dy}{dt} = f(y)$

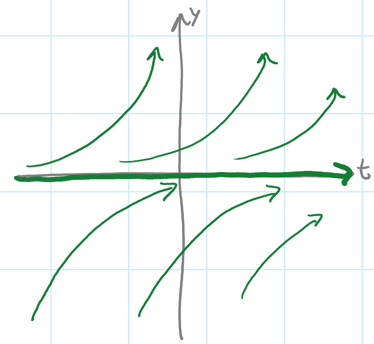
example: $\frac{dy}{dt} = y^2$ $\quad f(t,y) = y^2$



PHASE LINE

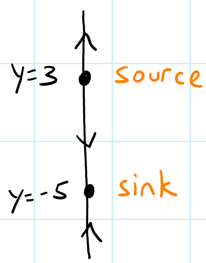


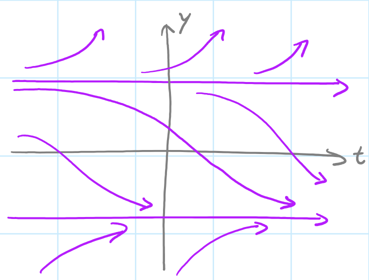
sketch solutions:



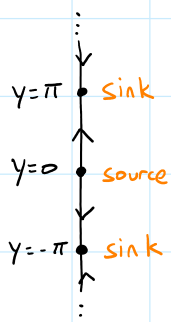
WORKSHEET

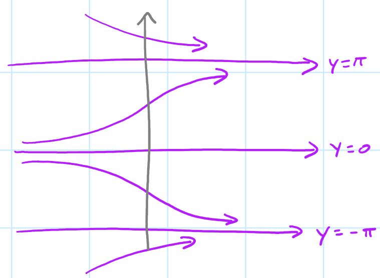
1. $\frac{dy}{dt} = (y-3)(y+5)$





2. $\frac{dy}{dt} = \sin(y)$

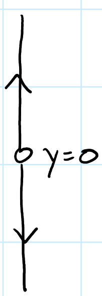


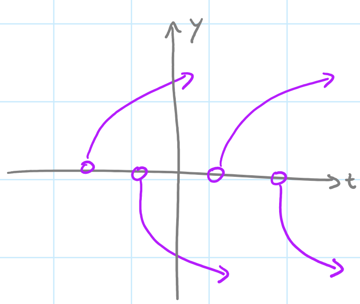


3. $\frac{dy}{dt} = \frac{1}{y}$

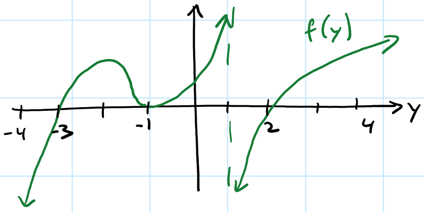
solution:

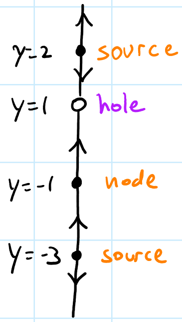
$y(t) = \pm\sqrt{t+c}$

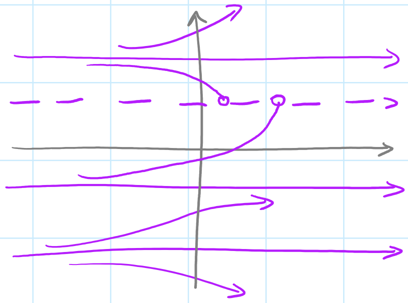




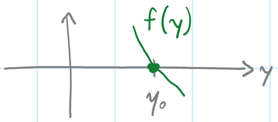
4. $\frac{dy}{dt} = f(y)$



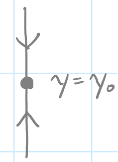




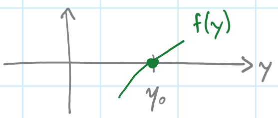
5. (a) $f'(y_0) < 0$



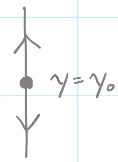
$f(y)$ goes from + to − at y_0, so the equilibrium point is a <u>sink</u>



(b) $f'(y_0) > 0$



$f(y)$ goes from − to + at y_0, so the equilibrium point is a <u>source</u>



(c) $f'(y_0) = 0$

Source, sink, and node are all possible!

Can you sketch examples for each of these cases?