Euler's Method Lab

Math 230

due Wednesday, September 9 at 4pm

Now that we have experience using Euler's method to approximate the solutions of differential equations, we will investigate the accuracy of Euler's method for two problems.

Use technology to help you answer the questions below. Be sure to answer the questions completely, typing your answers in a document. See below for details and grading information.

Problems

1. Consider the initial-value problem:

$$\frac{dy}{dt} = -(y-2)^2, \qquad y(0) = 4.$$

   (a) What are the equilibrium solutions of the differential equation? What does the existence and uniqueness theorem imply about the solution with $y(0) = 4$?

   (b) Use Euler's method with step size $\Delta t = 1$ to approximate $y(5)$. For this, compute in succession

$$(t_0, y_0), (t_1, y_1), \ldots, (t_5, y_5)$$

   where $t_0 = 0$, $y_0 = 4$ and $t_5 = 5$. List in table form the values you find for $(t_0, y_0), \ldots, (t_5, y_5)$. What goes wrong? What is the first y-value that is clearly a bad approximation?

   (c) Approximate $y(5)$ using Euler's method with step size 0.1. Then compute the exact solution (hint: separate variables) and find the value of $y(5)$. What is the error in your approximation? To find the error, use:

$$\text{error} = |\text{exact} - \text{approximation}|$$

   (d) Approximate $y(5)$ using Euler's method with step sizes 0.05, 0.01, 0.005, and 0.001. What are the corresponding errors?

   (e) Consider your answers to #2 and #3. How does the decrease in step size relate to the decrease in error? Specifically, if you decrease Δt by some factor, by approximately what factor does the error decrease?

   (f) Using your answer from part (e), what step size do you think you would you need in order to obtain an approximation correct to 12 decimal places?

2. Consider the initial-value problem:

$$\frac{dy}{dt} = -220y, \qquad y(0) = 2.$$

   (a) Use Euler's method with step size $\Delta t = 0.1$ to approximate $y(2)$. Look at the table of values (t_n, y_n). What do you observe? What do you conclude?

   (b) What happens if you use step size 0.01? What do you observe? What do you conclude?

   (c) What happens if you continue to decrease the step size? Can you choose a step size that results in an approximation that you consider to be good? Provide computational evidence that supports your answer.

3. What has this lab taught you about Euler's method?

Lab Report

Type your answers to the numbered items above in a document. You may use LaTeX a word processor (e.g. Microsoft Word), or Mathematica. If you use a word processor, please export your document as a PDF file. In your document, make sure you explain your work. A list of calculations without explanation will not suffice. For a few tips on writing solutions, see this helpful guide for mathematical writing. Use numbered headings in your document that correspond to the numbered items above, so that the professor can easily follow your work.

Grading

Your lab report will be graded out of 40 points. Problem 1 is worth 20 points, Problem 2 is worth 16 points, and Problem 3 is worth 4 points.

Points will be awarded based on the following criteria:

1. **Correctness**: Answers and supporting work are mathematically accurate.
2. **Completeness**: Assigned questions are answered completely, with appropriate details and justification to support the answers.
3. **Clarity**: Explanations are clear and concise. English sentences are used along with equations to explain mathematical reasoning.
4. **Presentation**: Work is presented in a typed document that is neat, organized, and easy to read.