INTEGRATING FACTOR METHOD

Solve $\frac{dy}{dt} + g(t)\,y = b(t)$

INTEGRATING FACTOR

$\mu(t) = e^{\int g(t)\,dt}$

$$\mu(t)\frac{dy}{dt} + \mu(t)g(t)\,y = \mu(t)\,b(t)$$

$$\int \frac{d}{dt}(\mu y)\,dt = \int \mu(t)\,b(t)\,dt$$

$$\mu y = \int \mu(t) b(t)\,dt$$

$$\boxed{y(t) = \frac{1}{\mu(t)}\int \mu(t)\,b(t)\,dt}$$

EXAMPLE 1:

$$\frac{dy}{dt} + \frac{2y}{t} = t$$

$g(t) = \frac{2}{t}$

Integrating Factor: $e^{\int \frac{2}{t}dt} = e^{2\ln(t)} = e^{\ln t^2} = \boxed{t^2 = \mu(t)}$

$$t^2\left(\frac{dy}{dt} + \frac{2y}{t}\right) = t^2 \cdot t$$

$$t^2\frac{dy}{dt} + 2ty = t^3$$

$$\int \frac{d}{dt}(t^2 y)\,dt = \int t^3\,dt$$

$$t^2 y = \frac{1}{4}t^4 + C$$

$$\boxed{y(t) = \frac{1}{4}t^2 + Ct^{-2}}$$

2.

$$\frac{dy}{dt} - 3t^2 y = e^{t^3}$$

Integrating factor: $\mu(t) = e^{\int -3t^2 dt} = e^{-t^3}$

Differential equation becomes: $\frac{d}{dt}\left(e^{-t^3}y\right) = 1$

General solution: $y(t) = (t+C)\,e^{t^3}$

3.

Integrating factor: $\mu(t) = e^{\int \frac{-2}{t+2}dt} = (t+2)^{-2}$

Differential equation becomes: $\frac{d}{dt}\left((t+2)^{-2}y\right) = t+2$

General solution: $y(t) = (t+2)^2\left(\frac{1}{2}t^2 + 2t + C\right)$