INTEGRATING FACTOR METHOD

Solve: $\frac{dy}{dt} + g(t)\,y = b(t)$

$$\mu(t)\frac{dy}{dt} + \mu(t)\,g(t)y = \mu(t)\,b(t)$$

Integrating Factor

$$\mu(t) = e^{\int g(t)\,dt}$$

$$\int \frac{d}{dt}(\mu\, y)\,dt = \int \mu(t)\,b(t)\,dt$$

$$\mu\, y = \int \mu(t)\,b(t)\,dt$$

$$\boxed{y(t) = \frac{1}{\mu(t)} \int \mu(t)\,b(t)\,dt}$$

EXAMPLE: $\frac{dy}{dt} + \frac{2y}{t} = t$

$$g(t) = \frac{2}{t}$$

$$t^2\left(\frac{dy}{dt} + \frac{2y}{t}\right) = t^2 \cdot t$$

Integrating factor: $\mu(t) = e^{\int g(t)\,dt} = e^{\int \frac{2}{t}\,dt}$

$$= e^{2\ln(t)} = e^{\ln(t^2)} = t^2$$

$$t^2\frac{dy}{dt} + 2yt = t^3$$

$$\frac{d}{dy}\left(t^2 y\right) = t^3$$

$$t^2 y = \int t^3\,dt = \frac{1}{4}t^4 + C$$

$$\boxed{y(t) = \frac{1}{4}t^2 + Ct^{-2}}$$

2. Integrating factor: $\mu(t) = e^{\int -3t^2\,dt} = e^{-t^3}$

Differential equation becomes: $\frac{d}{dt}\left(e^{-t^3}y\right) = 1$

Solution: $y(t) = (t + C)\,e^{t^3}$