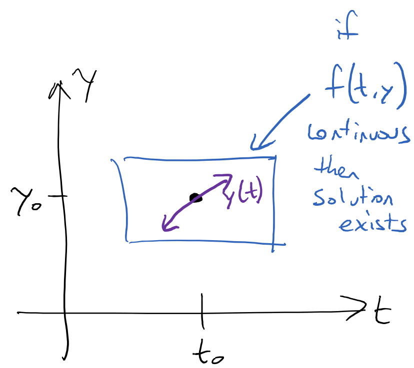
EXISTENCE AND UNIQUENESS

Situation: $\frac{dy}{dt} = f(t,y)$ and $y(t_0) = y_0$

- Does a solution exist?
- If so, is the solution unique?

THEOREM:



- If $f(t,y)$ is continuous in a rectangle containing (t_0, y_0), then a solution exists through this point. (Existence)

- If $f(t,y)$ and $\frac{\partial f}{\partial y}$ are both continuous in a rectangle containing (t_0, y_0), then the solution is unique in an interval around $t = t_0$. (Uniqueness)

PROBLEM 1:

$$\cos(t)\, y' - \sin(t)\, y = 3t \cos(t)$$

$$y' = \frac{3t\cos(t) + \sin(t)\, y}{\cos(t)}$$

$$y' = 3t + \tan(t)\, y \leftarrow f(t,y)$$

(a) If $t \neq \frac{\pi}{2} \pm n\pi$ $(n \in \mathbb{Z})$, then $f(t,y)$ is continuous and a solution $y(t)$ exists.

(b) $\frac{\partial f}{\partial y} = \tan(t)$ If $t \neq \frac{\pi}{2} \pm n\pi$, then $f(t,y)$ and $\frac{\partial f}{\partial y}$ are continuous, so the solution is unique.

(c) $y(0) = 0$ The solution is unique on the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\uparrow t_0$ $\uparrow y_0$

$(t_0, y_0) = (0,0)$