Euler's Method Lab

Math 230

due Wednesday, September 28 at 4pm

We now have some experience using Euler's method to approximate the solutions of differential equations. In this lab, we will investigate how the step size affects the accuracy of Euler's method.

You may (and should) use technology to help you answer the questions below. Be sure to answer the questions completely, typing your answers in a document. See below for details and grading information.

Part I

1. Let A be the day of the month on which you were born. For example, if you were born on February 16, then $A = 16$.

   Consider the initial-value problem

   $$\frac{dy}{dt} = -2t + 3, \qquad y(0) = A.$$

   Find the exact solution $y(t)$ to this problem and determine $y(1)$. Be sure to check that your answer here is correct and show this computation explicitly.

2. Use Euler's method with a step size of $\Delta t = 0.1$ to approximate $y(1)$. That is, compute in succession

   $$t_0, y_0, t_1, y_1, \ldots, t_{10}, y_{10}$$

   where $t_0 = 0$, $y_0 = A$ and $t_{10} = 1$, so that y_{10} is an approximation of $y(1)$. List in table form the values you find for t_0, $y_0, \ldots, y_{10}$. Highlight your approximate value for $y(1)$ using this step size. What is the error (the difference between your approximate value and the actual value of $y(1)$)?

3. Repeat item 2 with step size of $\Delta t = 0.05$ and twice as many steps.

4. Repeat item 2 with step size of $\Delta t = 0.01$ and ten times as many steps. You don't have to list all of the data here; just give the approximation to $y(1)$ that you find using this step size and the error.

5. In one or two paragraphs, discuss the improvement of the accuracy of Euler's method as you make the step size smaller by a factor of $\frac{1}{2}$ and $\frac{1}{10}$. How does this affect your approximation of $y(1)$? By what percent does the error decrease?

6. *Extra credit*: In items 2 through 4, notice that the error seems to depend on the step size in a predictable way. Formulate and prove a theorem about the error in the Euler's method approximation of $y(1)$, for the above initial-value problem, if the step size is $\Delta t = \frac{1}{N}$ for any positive integer N.

Part II

1. Let B be the number of the month in which you were born. For example, if you were born February, then $B = 2$.

   Consider the initial-value problem

   $$\frac{dy}{dt} = 2y + 3, \qquad y(0) = \frac{B}{10}.$$

   Find the exact solution $y(t)$ to this problem and determine $y(1)$. Be sure to check that your answer here is correct and show this computation explicitly.

2. Repeat items 2 – 5 above, approximating $y(1)$ using Euler's method with different step sizes. Then discuss your findings in one or two paragraphs.

Lab Report

Type your answers to the numbered items above in a document. You may use LaTeX or a word processor (e.g. Microsoft Word). *Do not* turn in a Mathematica file. Use numbered headings in your document that correspond to the numbered items above, so that the professor can easily follow your work.

Grading

Your lab report will be graded out of 40 points, based on the following criteria:

1. **Correctness**: Answers and supporting work are mathematically accurate. (20 points)
2. **Completeness**: Assigned questions are answered completely, with appropriate details and justification to support the answers. (8 points)
3. **Clarity**: Explanations are clear and concise. English sentences are used along with equations to explain mathematical reasoning. (6 points)
4. **Presentation**: Work is presented in a typed document that is neat, organized, and easy to read. (6 points)

The extra credit (Part I, #6) is worth up to 4 additional points.

based on a lab by Robert L. Devaney: `http://math.bu.edu/people/bob/MA226/euler-lab-s16.html`