Exam 1 Guidance

MATH 220 E • Spring 2025

The following is intended to help you focus your studying for Exam 1, which is on all the material we've studied in Chapters 1 and 2. The following might not encompass everything you could see on the exam, but it is intended to help you think about the course content and study effectively.

You should:

- Understand systems of equations from an algebraic perspective (i.e., symbols on the page) and a geometric perspective (intersecting lines or planes, etc.).
- Be able to find the solution set of a system of linear equations.
- Understand what a free parameter/free variable is and be able to explain the situation(s) in which they occur.
- Be able to write a solution set in "vector form."
- Understand how a system is related to an augmented matrix.
- Be able to **interpret** information about a system from an augmented matrix: whether the system is consistent or inconsistent, whether it has 0, 1, or infinitely many solutions.
- Understand what echelon and reduced row echelon form of a matrix are and what elementary row operations are.
- Be able to use row operations operations to reduce a matrix to echelon or reduced echelon form when needed.
- Understand and be able to work with vectors and their algebraic properties. You should also understand vectors geometrically (at least in $\mathbb{R}^2, \mathbb{R}^3$).
- Understand and be able to work with the notion of "linear combination."
- Know the definition of the span of a list of vectors (this is the "noun" version of span).
- Be able to determine if a given vector is or is not in the span of a list of vectors.
- Be able to determine whether or not a list of vectors spans $\mathbb{R}^n$ (this is the "verb" version of span).
- Have geometric intuition about span—visualize the VecMobile from the text!
- Understand Theorems 2.7 and 2.8 intuitively.
- Understand what the notation $A\mathbf{x} = \mathbf{b}$ means and that often we view this an equation where we want to solve for $\mathbf{x}$.
- Understand Theorem 2.11—it ties span, linear combination, systems of equations, and $A\mathbf{x} = \mathbf{b}$ notation together!

- Be good at rephrasing a question (about span, linear combination, vectors, etc) until you get a system of equations.
- Understand the definitions of linear independence and linear dependence of a list of vectors.
- Be able to determine when a list of vectors is linearly independent or linearly dependent.
- Be able to find a "dependence relation" when vectors are linearly dependent. Remember, this just means you should write one "nontrivial way to combine the vectors to get $\mathbf{0}$."
- Understand and be able to use Theorem 2.15—it's an alternative way to view linear dependence.
- Understand how linear independence is related to the equation $A\mathbf{x} = \mathbf{0}$ (i.e., a homogeneous system).
- Understand and be able to use Theorems 2.20 and 2.21.

What should I be able to do and what kind of (written) arguments am I expected to supply?

You should not have to write a paragraph for each problem—you'll never finish the exam if you do. Rather, a quick sentence explaining why you do what you do and why you conclude what you conclude is warranted. Don't let algebra speak for itself—you are in charge of convincing me that you know why you perform the steps you perform and that you understand what the result is telling you about the posed problem.

Can you give me an example of a well-written solution?

Sure. Suppose you need to determine if the vectors $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}$ span $\mathbb{R}^3$.

Here is a good solution:

We want to see if $A\mathbf{x} = \mathbf{b}$ is consistent for *all* vectors $\mathbf{b}$, where A is the matrix whose columns are the four vectors.

$$A = \begin{bmatrix}1 & 1 & 2 & 0\\1 & 2 & 3 & 1\\1 & 3 & 4 & 2\end{bmatrix} \xrightarrow{RREF} \begin{bmatrix}1 & 0 & 1 & -1\\0 & 1 & 1 & 1\\0 & 0 & 0 & 0\end{bmatrix}$$

(found via scratch work or calculator)

The third row is all 0, so the system $A\mathbf{x} = \mathbf{b}$ is not always consistent (there are some vectors $\mathbf{b}$ where the system will be inconsistent). So the columns of A do not span $\mathbb{R}^3$.

Notice how this explains WHY A was reduced and WHY the row of all 0's means that the vectors don't span $\mathbb{R}^3$. You need to say these things for full credit.

What can I use on the exam?

- You may use one 4×6-inch card of notes (one side) prepared in advance.

- The exam will involve only minimal row-reduction and no tedious arithmetic. Calculators are *not necessary*. However, you may use a calculator, but only for row-reducing matrices or simple arithmetic, and you must state where exactly you used your calculator. You may not use a phone or computer.

How should I study?

First, understand that people learn differently and process information in different ways and at different speeds. I suggest:

- Read through each section again and think about whether or not the main theorems make intuitive sense. Can you explain them out loud?

- Do a few problems each day, ramping up as we get closer to Friday. Talk with your classmates about the problems. Talk with me and visit the help sessions if you want to make sure things are correct or if you want to chat.

- Work on fluency! The exam is timed and you want to be able to do some of the problems efficiently. Perhaps give each other a few selected problems from a few different sections and time yourselves. This way, you won't know which section the problem came from.

- Look at the True/False questions and treat them as "True, Sometimes False, Always False" questions. If something is False, see if you can find an example of why. Is the statement *always* False or is it just False sometimes? Play with specific examples to convince yourself. These can be the BEST way to see if the theory is making sense to you.

- COME VISIT ME AND ASK QUESTIONS.