Linear Algebra – Day 25

MATH 220

1. Find $\det(A)$ if $A = \begin{bmatrix} 2 & 5 & -1 \\ 4 & 0 & 3 \\ 1 & 0 & 6 \end{bmatrix}$.

2. Find $\det(A)$ if $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 3 & 1 & 8 \\ 6 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \end{bmatrix}$

3. (a) Find the determinant of each of the following matrices:

☞ Pick a row which will make the calculation the easiest.

$$\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}, \qquad \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}, \qquad \begin{bmatrix} a & b & c & d \\ 0 & e & f & g \\ 0 & 0 & h & i \\ 0 & 0 & 0 & j \end{bmatrix}$$

(b) **Theorem:** If A is a upper-triangular square matrix, then $\det(A)$ equals __________________.

(c) What if A is a lower-triangular square matrix?

(d) What if A is a diagonal square matrix?

(e) What is the determinant of the identity matrix I_n?

☞ 1s on the diagonal...

4. Find the determinant of each of the following matrices. You should be able to do this *without* the recursive definition (i.e., cofactor expansion). Explain.

$$A = \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 4 & 1 & 0 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 & 6 & 2 \\ 3 & 7 & 23 & -2 \\ 2 & 1 & 6 & 2 \\ \pi & e & \sqrt{17} & 0.02 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 3 & 0 & 3 \\ 2 & 3 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

5. Determine whether each statement is true or false, and explain your reasoning.

Gather evidence by calculating some small, specific examples. Remember, evidence is not proof that something is TRUE. But, evidence can show something is FALSE.

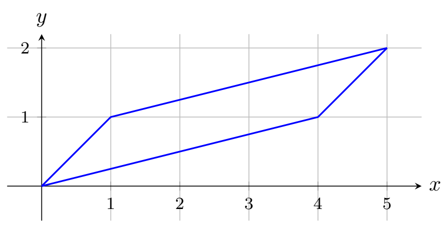
(a) $\det(A+B)$ equals $\det(A)+\det(B)$.

(b) $\det(A^T)$ equals $\det(A)$.

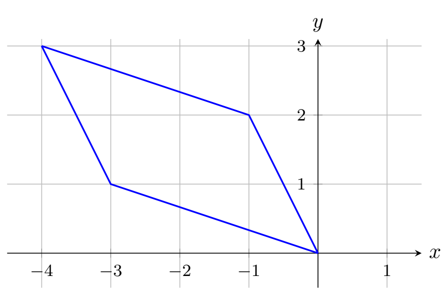
(c) $\det(AB)$ equals $\det(A)\det(B)$.

(d) $\det(A^{-1})$ equals $\det(A)$

6. Let $A = \begin{bmatrix} 4 & 1 \\ 1 & 1 \end{bmatrix}$. Find $\det(A)$. The diagram shows the image of the unit square in first quadrant *after* being multiplied by A. What is the area of the parallelogram?



7. Let $B = \begin{bmatrix} -3 & -1 \\ 1 & 2 \end{bmatrix}$. Find $\det(B)$. The diagram shows the image of the unit square in first quadrant *after* being multiplied by B. What is the area of the parallelogram?



8. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$. What is the connection between $\det(A)$ and the image of the unit square under T?

If you aren't sure, experiment with more 2×2 matrices!