Exam 2 Guidance

MATH 220 E • Spring 2025

The following is intended to help you focus your studying for Exam 2. Overall, you should know the material we have covered through Monday, March 24. While this exam will emphasize material since the last exam, this builds on what we studied in the first few weeks of the couse, so it's clearly necessary to know all the material well.

More specifically, the following list gives things you should know. I don't claim for this list to be exhaustive, but it should help you organize your studying. If you have questions about this list or about any topics, *now* is the time to formulate and ask those questions! I want to help.

You should:

- Understand what linear transformations are and be able to state the definition from Section 3.1. Also be able to give examples linear transformations and show a transformation is in fact linear.
- Understand how matrix multiplication is a linear transformation.
- Understand why all linear transformations are really matrix multiplication.
- Be able to find the matrix that performs a given linear transformation if the transformation is described in another way.
- Be able to interpret (some) linear transformations geometrically. In particular, you should be OK with rotations, dilations, contractions, and reflections. You should be able to interpret what a linear transformation does to a given region in the xy-plane or 3D space ($\mathbb{R}^2$, $\mathbb{R}^3$).
- Know what the range of a linear transformation is and how to interpret it as a span.
- Understand what one-to-one and onto mean, and be able to determine whether a linear transformation is one-to-one or onto.
- You should understand how everything from Chapters 1 and 2 ties into and informs Chapter 3. Yes, that means going back to things that you maybe didn't understand earlier, such as linear independence, span, and the equation $A\mathbf{x} = \mathbf{b}$ — these are all *incredibly* important notions throughout the course.
- Be able to work with all these definitions, methods, theorems and connections *fluently*. It's not enough to "know" them. You should have studied enough (and enough in advance) to develop good *intuition and instincts* about these problems. You will know how prepared you are if you can be asked a question and pretty quickly figure out what a solution path might be. You should not need your textbook at all by the time you finish your studying for the exam.
- You should know how to do algebra with matrices: multiply them, add them, multiply them by scalars, etc. You should understand and be able to use the properties of matrix addition and multiplication. You should know about the identity, the zero matrix, transpose, upper-triangular matrices, diagonal matrices (basically, lots of new terminology here but no crazy mathematical results). You *do not* need to know about "partitioned matrices" (even though they are in Section 3.2).

- You should understand how matrix multiplication is the result of composition of linear transformations and be able to use this fact to get a matrix that performs a given transformation if you can break that transformation into multiple steps.
- You should know what the inverse of a matrix *is* and *how* to calculate the inverse of a given matrix. You should understand how the inverse of a matrix A relates to the linear transformation T given by A and the inverse of T.
- You should know what a Markov Chain is. You should be able to interpret such a problem mathematically and model it with a transition matrix and an initial state vector. You should know how to find the long term (i.e., steady-state) behavior of a Markov Chain and you need to be able to find it *without* taking a "big" power of a matrix. Please remember that when constructing your matrix, you need to keep the states in the same order at all times.
- You should know the definition of a subspace of $\mathbb{R}^n$. If you are given a set of vectors S that have a certain common *property* or a common *form*, you should be able to prove that S is a subspace using the definition or prove that S is not a subspace (using a counterexample) with the definition.
- You should know that all spans are subspaces. If you are given a set S of vectors that have a certain common property or form, you should be able to prove S is a subspace by figuring out that S is a span and then using this fact. This way is often much faster and easier than the method mentioned in the previous line. This method cannot be used, however, when S is not a subspace.
- You should understand why the null space, kernel, and the range of a transformation are subspaces and how those concepts are related to the corresponding matrices.
- You should know what it means to have a basis for a subspace.
- You should be able to find a basis for a subspace S if you are given a description of S (first write S as a span and then figure out linear independence). In particular, you should be able to find a basis for the null space of a matrix. You should understand the inner-workings of Theorems 4.11 and 4.12: what do they mean, how are they used, etc..
- You should know the definition of dimension and be able to use it to describe subspaces of $\mathbb{R}^n$.
- You should understand the connections between the various parts of the *Unifying Theorem* and be able to explain these connections.

Problems

Here are a few problems that I think you should be readily able to solve and explain, without much hesitation. Use these however you want. They do not represent every kind of question you may be asked.

- **Section 3.1** #7, 23, 25, 27
- **Section 3.2** #13, 48
- **Section 3.3** #11, 13, 23

- **Section 4.1** #1-14 using the definition of subspace. For those subsets that are subspaces, you should also be able show this using the "span" theorem (Theorem 4.2), and you should be able to find a basis for each.
- **Section 4.1** # 25, 31, 33, 35
- **Section 4.2** # 7, 9, 13, 15, 31
- **All sections:** As many of the TRUE/FALSE exercises as you can do. Treat them all as *always*, *sometimes*, or *never* true questions.
- **All sections:** As many of the "find an example" exercises as you can do.

What can I use on the exam?

- You may use one 4×6-inch card of notes (one side) prepared in advance.
- The exam will involve only minimal row-reduction and no tedious arithmetic. Calculators are *not necessary*. However, you may use a calculator, but only for row-reducing matrices or simple arithmetic, and you must state where exactly you used your calculator. You may not use a phone or computer.

How should I study?

First, understand that people learn differently and process information in different ways and at different speeds. I suggest:

- Read through each section again and think about whether or not the main theorems make intuitive sense. Can you explain them out loud?
- Do a few problems each day, ramping up as we get closer to the exam day. Talk with your classmates about the problems. Talk with me and visit the help sessions if you want to make sure things are correct or if you want to chat.
- Work on fluency! The exam is timed and you want to be able to do some of the problems efficiently. Perhaps give each other a few selected problems from a few different sections and time yourselves. This way, you won't know which section the problem came from.
- COME VISIT ME AND ASK QUESTIONS.