Linear Algebra – Day 17

MATH 220

1. Let H be the collection of all vectors of the form $\begin{bmatrix} s \\ t \\ 0 \end{bmatrix}$ where s and t are real numbers.

👉 "Of the form" means "what the vectors look like."

(a) Just by looking at a vector, how can you tell whether or not the vector is in H?

(b) Is the vector $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ in H ?

> **Question #1:** If $\mathbf{u}$ is *any* vector in H and $\mathbf{v}$ is *any* vector in H, then will it always be true that $\mathbf{u} + \mathbf{v}$ is also a vector in H?
>
> **Question #2:** If $\mathbf{u}$ is *any* vector in H and c is *any* scalar, then it will always be true that $c \cdot \mathbf{u}$ is also a vector in H?

(c) **Sundar:** The answer to Question #1 is "yes."

Marissa: Correct! To prove that, you just need to explain why $\begin{bmatrix} a_1 \\ b_1 \\ 0 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ 0 \end{bmatrix}$ is in H.

Group chat: What is Marissa talking about? Is it true that $\begin{bmatrix} a_1 \\ b_1 \\ 0 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ 0 \end{bmatrix}$ is in H? Why?

(d) **Group chat:** The answer to Question #2 is also "yes." What would you have to do in order to *show* that?

(e) **Leo:** I just noticed that H equals span $\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right)$.

Group chat: Is Leo correct?

(f) **Sundar:** Wow! H equals $\mathbb{R}^2$.
Nadia (shaking head sadly): Oh, Sundar...
Group Chat: Why is Nadia sad? Or, is Sundar correct?

2. Let S be the collection of vectors in $\mathbb{R}^3$ of the form $\begin{bmatrix} x \\ x+1 \\ y \end{bmatrix}$.

Marissa: S is *not* a subspace of $\mathbb{R}^3$.

Group chat: Is Marissa correct? Investigate all three requirements for a subspace to see which one(s) go wrong.

- Is the zero vector, $\mathbf{0}$, in S?
- Can you find two vectors that are both in S, but their sum is NOT in S?
- Can you find a vector in S but some scalar multiple of that vector is NOT in S?

3. Let K be the collection of vectors in $\mathbb{R}^3$ of the form $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ where $x = 4z$ and $y = -3z$. Write K as a span of a single vector $\mathbf{d}$. Why does that show that K *is* a subspace of $\mathbb{R}^3$?

☞ Here, the "form" is not already "inside of" the vector. YOU have to put it there yourself.

4. Group chat: Is the set that *only contains the zero vector* (i.e., $\{\mathbf{0}\}$) a subspace of $\mathbb{R}^n$?

5. Group chat: Is $\mathbb{R}^n$ a subspace of $\mathbb{R}^n$?

6. Spicy! Does the collection of vectors of the form $\begin{bmatrix} a+b+2 \\ a-b-2 \\ 0 \end{bmatrix}$ a subspace of $\mathbb{R}^3$? Explain.

☞ Is this collection the span of some vectors?