Linear Algebra – Day 15

MATH 220

1. Warm up: Find the solution(s) to the following:

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

☞ These are just linear systems. How do we solve these, again?

(b) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

2. Simon: I could have solved (a) and (b) simultaneously by row reducing *one* 2×4 matrix! Wow!

Group chat: What does Simon mean?

3. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$.

Solve the matrix equation $A\mathbf{x} = \mathbf{b}$ by multiplying each side of the equation by A^{-1}.

4. Cleo: Hey Milo! I want to solve this for the mystery matrix X:

$$AX = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Milo: I get it! Let's try something similar to part (a) and multiply both sides by A^{-1}.

Cleo: OK! So when we do that, the A on the left gets canceled and we are left with

$$X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} A^{-1}$$

Milo (shaking head sadly): Oh, Cleo...

Group Discussion: Why is Milo sad?

5. Suppose $M = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $N = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

(a) Is matrix M *invertible*? In other words, does M^{-1} exist? Why or why not?

(b) How about matrix N? Is it invertible? Why or why not?

6. Suppose $C = \begin{bmatrix} 1 & 5 \\ 3 & 12 \end{bmatrix}$. Find C^{-1} by reducing the "augmented" matrix $\left[\begin{array}{c|c} C & I \end{array}\right]$.

7. Suppose $D = \begin{bmatrix} 1 & 3 & 2 \\ -1 & -7 & 6 \\ 0 & 4 & -8 \end{bmatrix}$. Try reducing the "augmented" matrix $\left[\begin{array}{c|c} D & I \end{array}\right]$. What happens? What does this tell you about matrix D?

8. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$.

(a) Compute $(AB)^{-1}$.

☞ first compute AB, and then find the inverse of the product

(b) Compute $A^{-1}B^{-1}$ and also $B^{-1}A^{-1}$.

☞ first compute A^{-1} and B^{-1} separately, and then multiply the two inverses.

9. (Challenge) Suppose that $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find A^{-1} by reducing the "augmented" matrix $\left[\begin{array}{c|c} A & I \end{array}\right]$ by hand.

☞ VARIABLES! YAY! Do your thing, don't let them scare you.