Linear Algebra – Day 13

MATH 220

1. The 4 matrices below are associated with the linear transformations, T_A, T_B, T_C, and T_D.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

(a) Determine which of the linear transformations is *onto* by determining if every vector in the codomain of the transformation is in the span of the columns of the corresponding matrix.

☞ RREF! Use Mathematica!

(b) Determine which of the linear transformations is one-to-one by determining how many pre-images of $\mathbf{0}$ there are.

2. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ with $m < n$, so the corresponding matrix has fewer rows than columns. Is the transformation *always*, *sometimes*, or *never* one-to-one? Explain.

☞ Look at T_C

3. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ with $m > n$, so the corresponding matrix has more rows than columns. Is the transformation *always*, *sometimes*, or *never* onto? Explain.

☞ Look at T_D

4. Consider the transformation T_A above.

(a) Describe the set of pre-images of $\mathbf{0}$ in vector form. (Remember that this is the set of solutions to $A\mathbf{x} = \mathbf{0}$.)

(b) Describe the set of pre-images of $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in vector form (i.e., the set of solutions to $A\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$).

(c) What is the geometric relationship between the two solution sets above?

5. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ -2 & 0 \\ -1 & 3 \end{bmatrix}$, and $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$.

(a) Which of the sums $A + B$, $A + C$, and $B + C$ can you compute?

(b) Compute $A + B$, $2A$, and $C + D^T$.

(c) Which of the products AB, AC, AD, CA, CB and CD can you compute?

(d) Compute AB by hand, and BA using Mathematica — use `B.A` rather than `B*A`. In fact, what does `B*A` give you?

6. Suppose $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 2 & -1 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & -2 & 3 \end{bmatrix}$.

Let T_A and T_B be the linear transformation given by multiplication by A and B respectively.

(a) Does $T_B \circ T_A$ even make sense?

☞ That is, is it possible to perform T_A first, immediately followed by T_B?

If yes, the domain of $T_B \circ T_A$ is ______ and the codomain is ______ .

(b) Does $T_A \circ T_B$ even make sense?

☞ That is, is it possible to perform T_B first, immediately followed by T_A?

If yes, the domain of $T_A \circ T_B$ is ______ and the codomain is ______ .

(c) You should have concluded that $T_A \circ T_B$ is the one that makes sense. Find the matrix that performs the transformation $T_A \circ T_B$.

☞ Hint: what happens to the "e" vectors?

(d) Compute AB. What do you notice?