Linear Algebra – Day 7

MATH 220

1. (a) **Erez:** I just noticed that the RREF of $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$.

Cleo: That means that $x_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + x_3 \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has a nontrivial solution!

Group chat: Is Cleo correct? How many solutions does this equation have?

(b) **Maura:** That means you can write $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$!

Group chat: Is Maura correct? How do you know?

(c) **Milo:** That means that $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ is in span $\left(\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right)$!

Group chat: Is Milo correct? How do you know?

2. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{u}_4 = \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix}$

(a) Does $x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 = \mathbf{0}$ have a non-trivial solution? If so, how many?

(b) Is the set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ linearly independent or dependent?

(c) Is $\mathbf{u}_1 \in \text{span}(\mathbf{u}_2, \mathbf{u}_3)$?

(d) Show that $\mathbf{u}_4 \in \text{span}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ and that there is only one way to write $\mathbf{u}_4$ as a linear combination of the others.

3. Let $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, and $\mathbf{u}_4 = \begin{bmatrix} 5 \\ 4 \\ 9 \end{bmatrix}$

(a) Does $x_1\mathbf{u}_1 + x_2\mathbf{u}_2 + x_3\mathbf{u}_3 = \mathbf{0}$ have a non-trivial solution? If so, how many?

(b) Is the set $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ linearly independent or dependent?

(c) Show that $\mathbf{u}_4 \in \text{span}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ and that there are infinitely many ways to write $\mathbf{u}_4$ as a linear combination of the others.

(d) Find two different ways to write $\mathbf{u}_4$ as a linear combination of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.

4. Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a set of three mystery vectors in $\mathbb{R}^3$. In each case below, (i) decide whether the set is linearly independent or dependent, and (ii) decide whether the set spans all of $\mathbb{R}^3$.

(a) The matrix $\begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$ has RREF $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) The matrix $\begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$ has RREF $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$.