Linear Algebra – Day 2

MATH 220

EVENTUAL GOAL: We want to *eventually* (not now) solve the system

$$\begin{aligned} x + 2y + 3z &= 4 \\ 2x + 4y + 8z &= 10 \\ x + 3y + 4z &= 6 \end{aligned}$$

Quick Group Discussion: If you were going to try to solve this system right NOW, what strategies might you try?

☞ DO NOT actually do it.

1. If we were to graph each of the three equations above, they would each be a *plane* in 3 dimensions.

Milo: Hey Ava. I think it's really cool that in 2 dimensions, two lines can either be parallel, intersect in exactly one point, or be the same line.

Ava: That is cool! I wonder what happens with two planes in 3 dimensions.

Group chat: Investigate! How could two planes interact in 3 dimensions?

☞ Try drawing or using your hands or...

Milo: That was cool. Now what about THREE planes! How could THREE planes interact?

Group chat: Well? What do you think?

2. Find all the solutions for the following system.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ x_2 + x_3 &= 2 \\ x_3 &= 1 \end{aligned}$$

- This system is (circle one) consistent / inconsistent.
- The solution set (circle one) has a free variable / doesn't have a free variable.

3. **Ava:** Hello, Jason. Did you notice that the system at the top of the page and the system in #2 both involve three equations and three unknown variables?

Jason: Wassup, Ava? So, this means the two systems are equally difficult to solve.

Group discussion: What is your response to Jason?

☞ Be nice.

4. Here are three systems of equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ x_2 + x_3 &= 2 \end{aligned} \qquad \begin{aligned} x_1 + x_2 + x_3 + x_4 &= 1 \\ x_2 - x_3 \quad &= 2 \\ x_4 &= 3 \end{aligned} \qquad \begin{aligned} x_1 + x_2 + x_3 + x_4 &= 1 \\ x_3 + x_4 &= 3 \end{aligned}$$

☞ You'll need one free parameter/variable for (a) and (b), and two for (c).

(a) **Group discussion:** Why are each of these easier to solve than the system at the top of page #1?

(b) **Group discussion:** Try actually finding the solutions to each system.

(c) **Ava:** Milo! I can tell JUST BY LOOKING at each of these systems in #4 *exactly* how many free variables there will be!! WOW!!

Group chat: Can you figure why Ava knows right away how many free variables there are going to be?

5. Below are steps that Jason used to solve a system. To the *right* of each step, write down the one thing Jason did, if you can figure it out!

START:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 2x_1 + 4x_2 + 8x_3 &= 10 \\ x_1 + 3x_2 + 4x_3 &= 6 \end{aligned}$$

STEP 1:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 2x_1 + 4x_2 + 8x_3 &= 10 \\ x_2 + x_3 &= 2 \end{aligned}$$

STEP 2:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ 2x_3 &= 2 \\ x_2 + x_3 &= 2 \end{aligned}$$

STEP 3:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ x_2 + x_3 &= 2 \\ 2x_3 &= 2 \end{aligned}$$

STEP 4:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ x_2 + x_3 &= 2 \\ x_3 &= 1 \end{aligned}$$

STEP 5: NOW back-substitution works! What's the solution to the system?