

7 Feb. 2025

# Linear Algebra – Day 1

MATH 220

## Scenario

Scientists have been watching the spread of happiness this year. There are three different moods people can be in: Unhappy ( $N$ ), Happy and they know it ( $H$ ), and Happy without realizing ( $R$ ). Each week, the same 1000 people are checked for happiness:

- **Of those who are currently unhappy:** Next week, 50% will become happy and know it and 25% will become happy without realizing.
- **Of those currently happy and they know it:** Next week, 50% will become unhappy and 10% will remain happy but not realize it anymore.
- **Of those currently happy without realizing:** Next week, 10% will become unhappy and 10% will now know they are happy.

## Discuss these questions with your group

1. Today's check revealed 600 unhappy people and 400 happy people (but only 100 know they are happy).

(a) Next week, how many people does the model predict will be unhappy? How many will be happy and know it?

$$\begin{aligned} \text{unhappy:} & \quad \frac{1}{4}(600) + \frac{1}{2}(100) + \frac{1}{10}(300) = 230 \\ \text{happy and know it:} & \quad \frac{1}{2}(600) + \frac{2}{5}(100) + \frac{1}{10}(300) = 370 \\ \text{happy without realizing:} & \quad \frac{1}{4}(600) + \frac{1}{10}(100) + \frac{4}{5}(300) = 400 \end{aligned}$$

(b) What about two weeks from now?

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{unhappy} & \text{happy} & \text{happy not} \\ & \text{know it} & \text{realizing} \end{array}$$

👉 This is a "model." Go ahead and round your answers. There are no fractional people!

(c) Can you predict what will happen one year from now?

2. Suppose that

$n_i$  = the number of unhappy people  $i$  weeks from today

$h_i$  = the number of happy and they know it people  $i$  weeks from today

$r_i$  = the number of happy without realizing people  $i$  weeks from today

With your group, come up with some equations that can help explain what's happening each week.

$$\begin{aligned} \overbrace{\frac{1}{4}n_i + \frac{1}{2}h_i + \frac{1}{10}r_i}^{\text{week } i} &= \overbrace{n_{i+1}}^{\text{week } i+1} \\ \frac{1}{2}n_i + \frac{2}{5}h_i + \frac{1}{10}r_i &= h_{i+1} \\ \frac{1}{4}n_i + \frac{1}{10}h_i + \frac{4}{5}r_i &= r_{i+1} \end{aligned}$$

👉 For example, try to write  $h_{i+1}$  in terms of  $n_i$ ,  $h_i$ , and  $r_i$ .

We didn't have time to discuss the following in class.

Feel free to think about them or ask about them in office hours!

3. The first happiness check was incorrectly done! Today's check actually revealed 100 unhappy people and 900 happy people (and 800 of them know it).

(a) Now how many people will be happy next week? ...unhappy?

(b) What do you think will happen one year from now?

4. Suppose that an oracle tells you that *next week*, a particular group of people will have 500 unhappy people and 760 happy people who know it. In this case, how many people in the group *this week* could be unhappy, and how many could be happy and know it?

☞ Not enough information? Can you make an assumption?