Exam 2 Review Problems

Math 126

1. One of the following integrals can be evaluated via u-substitution, but the other requires integration by parts. Which is which?

$$\int xe^x\,dx \qquad\qquad \int xe^{x^2}\,dx$$

2. Evaluate $\int x\sin(x)\,dx$.

3. Evaluate each integral or show that it is divergent:

   (a) $\displaystyle\int_0^\infty \frac{x}{e^x}dx$

   (b) $\displaystyle\int_1^2 \frac{dx}{x\ln(x)}$

4. Use the arc length formula to find the length of the curve $y = mx$, where m is a constant, from $x = 0$ to $x = 4$. Then use geometry to check you answer.

5. Find the length of the curve given by $y = \frac{x^4}{8} + \frac{1}{4x^2}$ for $1 \le x \le 3$.

   Hint: $1 + \left(\frac{x^3}{2} - \frac{1}{2x^3}\right)^2 = \left(\frac{x^3}{2} + \frac{1}{2x^3}\right)^2$

6. Find the centroid of the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$.

7. A demand curve is given by $p = 600/(x + 50) + 10$. Find the consumer surplus when the selling price is \$16.

8. (a) What is a differential equation?

   (b) What is a separable differential equation?

9. A population size is modeled by the differential equation

$$\frac{dP}{dt} = 2P\left(1 - \frac{P}{6000}\right)$$

   (a) For what values of P is the population size increasing?

   (b) For what values of P is the population size decreasing?

   (c) What are the equilibrium solutions?

10. Suppose that the graph of $y(x)$ passes through the point $(0, 1)$ and the slope at (x, y) is xy.

    (a) Write a differential equation satisfied by $y(x)$.

    (b) Solve the differential equation for $y(x)$. (Don't forget the initial value!)

11. Evaluate the integral or show that it is divergent:

$$\int_2^\infty (x-1)^{-3/2}\,dx$$

12. Suppose the demand curve for a commodity is given by $p(x) = 18 - x$ and the supply curve is given by $p_S(x) = 2 + \frac{x}{3}$.
    (a) What is the market equilibrium price (that is, the price $P = p(x) = p_S(x)$)?
    (b) What is the consumer surplus?
    (c) What is the producer surplus?

13. Find the length of the curve given by $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = 1$ to $x = 3$.

14. Find $y(x)$ that satisfies $\frac{dy}{dx} = x^2(2-y)^2$ and $y(0) = 1$.

15. Sketch a slope field for the differential equation $\frac{dy}{dx} = \frac{x}{y}$. If a solution satisfies $y(0) = 1$, use your slope field to estimate $y(2)$.

16. Find the centroid of the region bounded by the x-axis, the y-axis, and the graph of the line $y = 4 - 2x$.

17. Find the solution to the differential equation $2t\frac{dy}{dt} + y = 4t^2 + 3t$ with $y(1) = 2$. (You may assume $t > 0$.)

18. *Challenge*: Evaluate $\displaystyle\int_1^{e^\pi} \sin(\ln(x))\,dx$.

The following formulas will be provided on Exam 2:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

$$M_y = \int_a^b x(f(x) - g(x))\,dx \qquad M_x = \int_a^b \frac{1}{2}\left[(f(x))^2 - (g(x))^2\right]dx$$

$$C = \int_0^X [p(x) - P]\,dx \qquad S = \int_0^X [P - p_S(x)]\,dx$$

$$F = \frac{A}{\int_0^T c(t)\,dt} \qquad P(t) = \frac{M}{1 + Ae^{-kt}} \text{ where } A = \frac{M - P_0}{P_0}$$

Problems from the textbook, for more practice

- Section 7.1: #1, 3, 5, 7, 9, 11, 13, 15, 23, 27
- Section 7.8: #1, 2, 7ab, 11ab, 19, 31, 44
- Section 8.1: #1, 3, 5, 7, 9, 11, 13, 15
- Section 8.3: #21, 23, 25, 27, 29, 31
- Section 8.4: #1, 3, 5, 9, 17
- Section 9.1: #1, 3, 5, 7, 9, 13
- Section 9.2: #1, 3–6, 7, 11, 13
- Section 9.3: #1, 3, 5, 7, 9, 11, 13, 43
- Section 9.4: #1, 3, 5, 7, 9, 19
- Section 9.5: #1, 3, 5, 7, 9, 11, 13, 15, 17