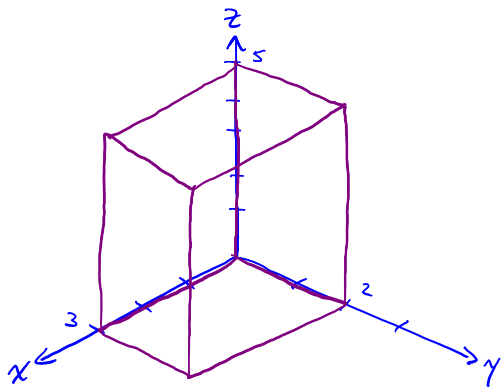
Double Integrals and Iterated Integrals

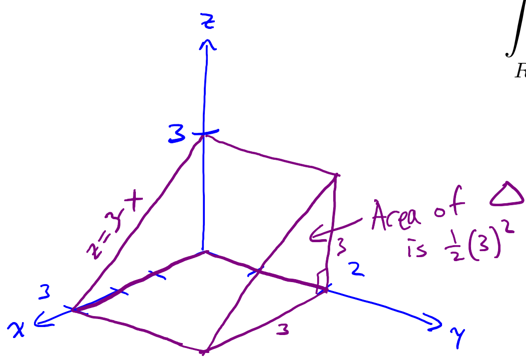
Sections 15.1 and 15.2

1. Sketch the solid under the graph of $f(x, y) = 5$ and above the region $R = \{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 2\}$. Then evaluate the following integral.



$$\iint_R 5\, dA = 2(3)(5) = 30$$

2. Sketch the solid under the graph of $f(x, y) = 3 - x$ and above the region $R = \{(x, y) \mid 0 \leq x \leq 3, 0 \leq y \leq 2\}$. Then evaluate the following integral.

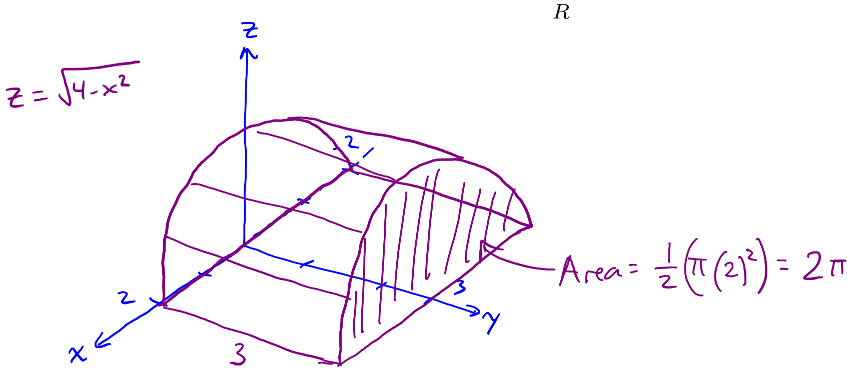


$$\iint_R (3 - x)\, dA$$

$$= \frac{1}{2}(3)^2(2) = 9$$

3. Sketch the solid under the graph of $f(x, y) = \sqrt{4 - x^2}$ and above the region $R = \{(x, y) \mid -2 \leq x \leq 2, 0 \leq y \leq 3\}$. Then evaluate the following integral.

$$\iint_R \sqrt{4 - x^2}\, dA = 3(2\pi) = 6\pi$$



ITERATED INTEGRAL

4. Evaluate $\int_0^3 \boxed{\int_1^2 x^2 y\, dx}\, dy$.

$$\int_1^2 x^2 \underset{\text{constant}}{y}\, dx = y\frac{x^3}{3}\Big|_{x=1}^{x=2} = y\cdot\frac{8}{3} - y\frac{1}{3} = \frac{7y}{3}$$

$$\int_0^3\int_1^2 x^2 y\, dx\, dy = \int_0^3 \frac{7y}{3}\, dy = \frac{7}{3}\cdot\frac{y^2}{2}\Big|_{y=0}^{y=3} = \frac{7}{3}\cdot\frac{9}{2} - \frac{7}{3}\cdot\frac{0}{2}$$

$$= \frac{7}{\cancel{3}}\cdot\frac{\cancel{9}3}{2} = \boxed{\frac{21}{2}}$$

Fubini's Theorem: If f is continuous on the rectangle $R = \{(x, y) \mid a \le x \le b, c \le y \le d\}$, then:

$$\iint_R f(x,y)\, dA = \int_a^b\int_c^d f(x,y)\, dy\, dx = \int_c^d\int_a^b f(x,y)\, dx\, dy$$

5. Evaluate $\iint_R x\cos(xy)\, dA$, where $R = \{(x, y) \mid 0 \le x \le \pi, 0 \le y \le 1\}$

$$\iint_R x\cos(xy)\, dA = \int_0^\pi\int_0^1 x\cos(xy)\, dy\, dx = \int_0^\pi \sin(x)\, dx = -\cos(x)\Big|_{x=0}^{x=\pi} = -\cos(\pi) + \cos(0)$$

$$= -(-1) + 1 = \boxed{2}$$

$$\int_0^1 x\cos(xy)\, dy = \sin(xy)\Big|_{y=0}^{y=1} = \sin(x\cdot 1) - \sin(x\cdot 0) = \sin(x)$$

6. Find the volume of the solid that lies under the graph of $z = x^2 + y$ and above the rectangle $R = [-1, 1] \times [0, 3]$.

$$\text{Volume} = \int_{-1}^1 \boxed{\int_0^3 (x^2+y)\, dy}\, dx = \int_{-1}^1 \left(3x^2 + \frac{9}{2}\right) dx = \left[x^3 + \frac{9}{2}x\right]_{x=-1}^{x=1} = \left(1+\frac{9}{2}\right) - \left(-1-\frac{9}{2}\right) = \boxed{11}$$

$$\int_0^3 (x^2+y)\, dy = \left[x^2 y + \frac{1}{2}y^2\right]_{y=0}^{y=3} = \left(3x^2 + \frac{9}{2}\right) - (0) = 3x^2 + \frac{9}{2}$$