The **gradient** of $f(x, y)$ is a vector, denoted ∇f, and defined by:

$$\nabla f = \left\langle f_x(x,y),\ f_y(x,y) \right\rangle$$

The maximum value of the directional derivative $D_{\mathbf{u}}f$ is $|\nabla f|$, and it occurs when $\mathbf{u}$ has the same direction as the gradient vector ∇f.

3. Find the gradient of $f(x, y) = x^2 + x\sin(2y)$.

$$\nabla f = \left\langle f_x(x,y),\ f_y(x,y) \right\rangle$$
$$= \left\langle 2x + \sin(2y),\ 2x\cos(2y) \right\rangle$$

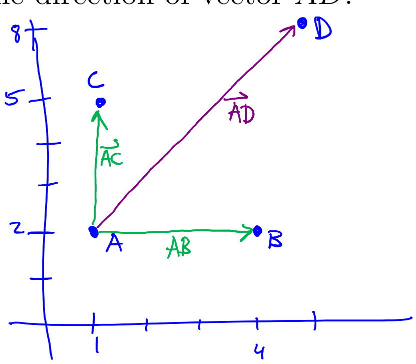
4. What is the direction of the maximum rate of change of $f(x, y) = x^2 + x\sin(2y)$ at the point $(2, 0)$? What is this rate of change?

$$\nabla f(2,0) = \left\langle 2(2) + \sin(2(0)),\ 2(2)\cos(2(0)) \right\rangle = \langle 4, 4 \rangle$$

direction of steepest slope: $\langle 1,1\rangle$ or $\langle 4,4\rangle$ or $\left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$

slope in this direction: $|\langle 4,4\rangle| = 4\sqrt{2}$

5. Let $f(x, y)$ have continuous partial derivatives and consider the points $A(1, 2)$, $B(4, 2)$, $C(1, 5)$, and $D(5, 8)$. The directional derivative of f at A in the direction of vector $\overrightarrow{AB}$ is 3, and the directional derivative at A in the direction of $\overrightarrow{AC}$ is 15. Find the directional derivative of f at A in the direction of vector $\overrightarrow{AD}$.



$$D_{\overrightarrow{AB}} f(1,2) = f_x(1,2) = 3$$

$$D_{\overrightarrow{AC}} f(1,2) = f_y(1,2) = 15$$

$$D_{\overrightarrow{AD}} f(1,2) = D_{\vec{u}} f(1,2) = \frac{2}{\sqrt{13}}(3) + \frac{3}{\sqrt{13}}(15) = \boxed{\frac{51}{\sqrt{13}}}$$

unit vector: $\vec{u} = \dfrac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \dfrac{\langle 4,6\rangle}{\sqrt{4^2+6^2}} = \dfrac{\langle 4,6\rangle}{\sqrt{52}} = \left\langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \right\rangle$

6. Explain why the directional derivative is really a dot product involving the gradient vector.

If $\vec{u} = \langle a,b\rangle$, then $D_{\vec{u}} f(x,y) = \langle a,b\rangle \cdot \langle f_x(x,y), f_y(x,y)\rangle = \vec{u}\cdot\nabla f$