Tangent Planes and Linear Approximations

Section 14.4

Suppose f has continuous partial derivatives. An equation of the **tangent plane** to the surface $z = f(x, y)$ at the point $P(x_0, y_0, z_0)$ is:

$$z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

1. Given that f is a differentiable function with $f(2, 5) = 6$, $f_x(2, 5) = 1$ and $f_y(2, 5) = -1$, use a linear approximation to estimate $f(2.2, 4.9)$.

Tangent plane to $f(x,y)$ at $(2,5,6)$.

$$z - z_0 = f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

$$z - 6 = 1\,(x-2) + -1\,(y-5)$$

$$z = x - y + 9 \quad \text{so} \quad f(x,y) \approx x - y + 9 \text{ near } \underset{(x,y)}{(2,5)}$$

Thus, $f(2.2, 4.9) \approx 2.2 - 4.9 + 9 = \boxed{6.3}$

2. Let $f(x, y) = 2x^2 + y^2$. Find the equation of the tangent plane to the surface $z = f(x, y)$ at the point $(1, 1, 3)$.

$f_x(x,y) = 4x \qquad f_x(1,1) = 4 \qquad x_0 = 1,\ y_0 = 1,\ z_0 = 3$

$f_y(x,y) = 2y \qquad f_y(1,1) = 2$

Tangent plane: $z - 3 = 4(x-1) + 2(y-1)$

$$z = \underbrace{4x + 2y - 3}$$

Use the tangent plane to approximate $f(.9, 1.1)$.

$$f(0.9, 1.1) \approx 4(0.9) + 2(1.1) - 3 = 2.8$$

3. The wave heights h in the open sea depend on the speed v of the wind and the length of time t that wind has been blowing at that speed (knots). Values of the function $h = f(v, t)$ are recorded in feet in the following table.

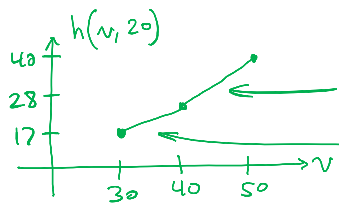
t

v

		Duration (hours)						
		5	10	15	20	30	40	50
Wind speed	20	5	7	8	8	9	9	9
	30	9	13	16	17	18	19	19
	40	14	21	25	28	31	33	33
	50	19	29	36	40	45	48	50
	60	24	37	47	54	62	67	69

(a) Use the table to find a linear approximation to the wave height function when v is near 40 knots and t is near 20 hours.

$$h_v(40,20) \approx \frac{1}{2}\left(\frac{12}{10} + \frac{11}{10}\right) = \frac{23}{20} \approx 1.15$$



$\text{slope} = \frac{40-28}{50-40} = \frac{12}{10}$

$\text{slope} = \frac{28-17}{40-30} = \frac{11}{10}$

$h_t(40,20) \approx ?$

to be continued...

(b) Using the linear approximation you just found, estimate the wave heights when the wind has been blowing for 24 hours at 43 knots.

4. Let $f(x, y) = \sqrt{y + cos^2(x)}$. Use a linearization to approximate $f(0.2, 0.1)$.