Differential Equations

Sections 9.1 and 9.2

1. Show that $y = \frac{1}{16}x^4$ is a solution to the differential equation $\frac{dy}{dx} = xy^{1/2}$.

$$\frac{dy}{dt} = \frac{1}{4}x^3 \quad = \quad xy^{1/2} = x\left(\frac{1}{16}x^4\right)^{1/2} = x\left(\frac{1}{4}x^2\right) = \frac{1}{4}x^3$$

since both sides are equal, $y = \frac{1}{16}x^4$ satisfies $\frac{dy}{dt} = xy^{1/2}$

2. Show that $y = xe^x$ is a solution to the differential equation $y'' - 2y' + y = 0$ with initial value $y(0) = 0$.

$$\left.\begin{aligned} y' &= xe^x + e^x \\ y'' &= xe^x + 2e^x \end{aligned}\right\}$$

$$y'' - 2y' + y = (xe^x + 2e^x) - 2(xe^x + e^x) + (xe^x)$$
$$= \cancel{xe^x} + \cancel{2e^x} - \cancel{2xe^x} - \cancel{2e^x} + \cancel{xe^x} = 0$$

Thus, $y = xe^x$ satisfies $y'' - 2y' + y = 0$

3. Suppose that the size of a population is modeled by the differential equation $\frac{dP}{dt} = 4P\left(1 - \frac{P}{3000}\right)$.

(a) For what values of P is the population increasing?

	0		3000	
P		$+$		$+$
$\left(1 - \frac{P}{3000}\right)$		$+$		$-$
$\frac{dP}{dt}$		$+$		$-$

P is increasing if $0 < P < 3000$

(b) For what values of P is the population decreasing?

P is decreasing if $P > 3000$

(c) What are the constant solutions (the "equilibrium" solutions) of the differential equation?

$P = 0$ and $P = 3000$

4. Which of the differential equations below corresponds to the direction field? How do you know?

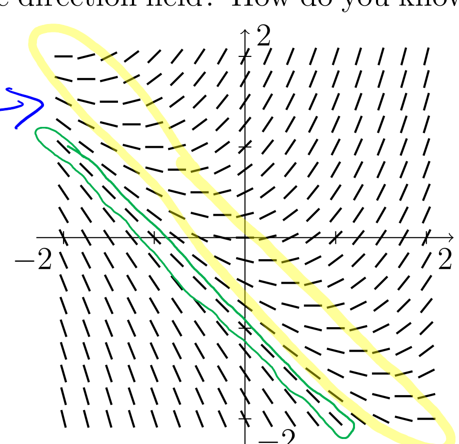
(a) $\frac{dy}{dx} = \sin(x)$

(b) $\frac{dy}{dx} = x^2 + y$

(c) $\frac{dy}{dx} = x - y$

(d) $\frac{dy}{dx} = x + y$ ← This is the only choice for which $\frac{dy}{dx} = 0$ for $x = -1$, $y = 1$.

Observe that slopes are zero along $y = -x$



looks like $y = -x - 1$ is a solution

5. Does the direction field in question 4 suggest any solutions to the differential equation? Guess a solution and check to see if it in fact solves the differential equation that you selected in 4.

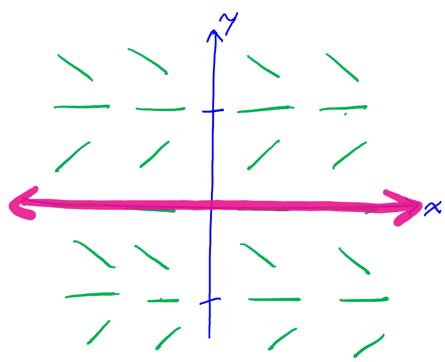
It appears that $y=-x-1$ may be a solution.

Check: If $y=-x-1$, then $\frac{dy}{dx}=-1$, which is the same as $x+y$.

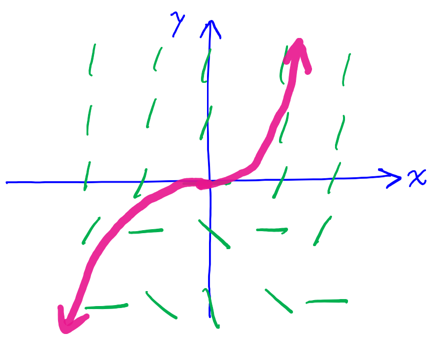
So yes, $y=-x-1$ satisfies $\frac{dy}{dx}=x+y$.

6. Sketch direction fields for each of the following differential equations. Then sketch the solution curve that passes through the origin.

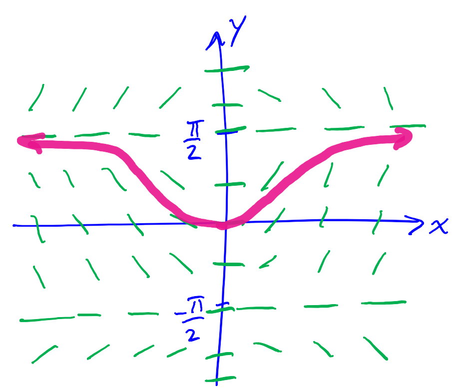
(a) $\dfrac{dy}{dx} = \sin(y)$



(b) $\dfrac{dy}{dx} = x^2 + y$



(c) $\dfrac{dy}{dx} = x\cos(y)$



(d) $\dfrac{dy}{dx} = \dfrac{x}{y+1}$