More Practice Problems

MATH 126

Integration

1. Estimate $\int_0^2 x^2 - x\,dx$ using a right-hand sum with $n = 4$ subintervals.

2. Evaluate $\int_0^1 x + \sqrt{1-x^2}\,dx$ without taking any antiderivatives.

3. A car is traveling down a road with velocity $v(t) = t^2 - t$ meters per second (t is measured in seconds).

   (a) How far from the starting point ($t = 0$) is the car after 5 seconds?

   (b) What is the *total* distance the car drove between $t = 0$ and $t = 5$?

4. Evaluate each integral.

   (a) $\int x^2 \cos(x^3)\,dx$

   (b) $\int x^2 \sin(x)\,dx$

   (c) $\int \frac{x+2}{x^2+4x}\,dx$

   (d) $\int \frac{x}{\sqrt{1-x^2}}\,dx$

   (e) $\int x^5 \ln(x)\,dx$

   (f) $\int \sin(x)\cos(\cos(x))\,dx$

   (g) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

   (h) $\int_0^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

   (i) $\int_e^\infty \frac{1}{x(\ln(x))^2}\,dx$

   (j) $\int_1^\infty \frac{x+1}{x\sqrt{x}}\,dx$

5. Find the area of the region bounded by $y = x^2$ and $y = 4x - x^2$.

6. Find the volume when region bounded by $y = x^2$ and $y = 4x - x^2$ is rotated about the x-axis.

7. Find the volume when region bounded by $y = x^2$ and $y = 4x - x^2$ is rotated about the line $y = -2$.

8. Find the volume when region bounded by $y = x^2$ and $y = 4x - x^2$ is rotated about the line $y = 6$.

9. Find the length of the boundary of the region bounded by $y = x^2$ and $y = 4x - x^2$.

Sequences and Series

1. Define what it means for a sequence $\{a_n\}$ to converge.

2. Define what it means for a series $\sum_{n=0}^{\infty} a_n$ to converge. (Your answer must involve the *sequence of partial sums.*)

3. Let $a_n = \frac{9^{n+1}}{10^n}$. Determine whether the sequence $\{a_n\}$ converges or diverges.

4. Let $a_n = \frac{9^{n+1}}{10^n}$. Determine whether the series $\sum_{n=0}^{\infty} a_n$ converges or diverges.

5. Find any example of a series that converges to π.

6. Does each series converge or diverge? Explain why.

   (a) $\sum_{n=0}^{\infty} \frac{n}{n^3 + 1}$

   (b) $\sum_{n=0}^{\infty} \frac{n^3}{5^n}$

   (c) $\sum_{n=0}^{\infty} \frac{n^2 + n - 1}{n^3 + 5n^2 - n + 2}$

   (d) $\sum_{n=0}^{\infty} \frac{n^3 + 5n^2 - n + 2}{n^2 + n - 1}$

   (e) $\sum_{n=0}^{\infty} \frac{(-5)^{2n}}{n^2 24^n}$

   (f) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

7. Find the interval of values of x for which $\sum_{n=2}^{\infty} \frac{(x+2)^n}{n4^n}$ converges. What is the radius of convergence?

8. Find the Maclaurin Series for $f(x) = \dfrac{1}{1-x^5}$.

9. Find the Maclaurin Series for $f(x) = \dfrac{x}{2+x}$.

10. Find the Maclaurin Series for $f(x) = x\sin(x^4)$.

11. Find $f^{(28)}(0)$ and $f^{(29)}(0)$ for $f(x) = x\sin(x^4)$.

12. Without taking derivatives, find the first three nonzero terms of the Maclaurin Series for $f(x) = e^x\cos(x)$.

13. Find the Taylor polynomial of degree 3, centered at $a = \pi/2$, for $f(x) = x\cos(x)$.

3D Coordinates, Vectors, Lines, and Planes

1. Find a vector of length 3 in the opposite direction of $\langle 3, 2, 6\rangle$.

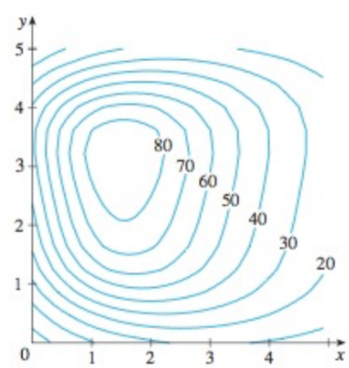
2. Find parametric equations for the line through $(4, -1, 2)$ and $(1, 1, 6)$.

3. Find parametric equations for the line through $(-2, 2, 4)$ and perpendicular to the plane $2x - y + 5z = 17$.

4. Find an equation for the plane through $(1, 2, -2)$ that contains the line $x = 2t, y = 3 - t, z = 1 + 3t$.

Derivatives and Integrals of 2-Variable Functions

1. The contour map of f is shown below.



   (a) Estimate the value of $f(3, 2)$.

   (b) Is $f_x(3, 2)$ positive or negative? Explain.

   (c) Which is greater, $f_y(2, 1)$ or $f_y(2, 2)$? Explain.

2. A metal plate is situated in the xy-plane and occupies the rectangle $0 < x < 10$, $0 < y < 8$, where x and y are measured in meters. The temperature at the point (x, y) in the plate is $T(x, y)$, where T is measured in degrees Celsius. Temperatures at equally spaced points were measured and recorded in the table.

x \ y	0	2	4	6	8
0	30	38	45	51	55
2	52	56	60	62	61
4	78	74	72	68	66
6	98	87	80	75	71
8	96	90	86	80	75
10	92	92	91	87	78

(a) Estimate the values of the partial derivatives $T_x(6,4)$ and $T_y(6,4)$. What are the real life meanings of these two numbers? Use units.

(b) Estimate the value of $D_{\mathbf{u}}T(6,4)$, where $\mathbf{u} = \langle 1, 1\rangle$. Interpret your result.

3. Calculate $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for $z = xe^{xy}$.

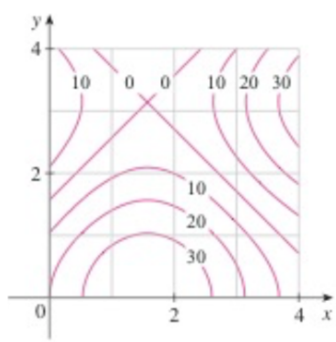
4. Find the equation of the plane that is tangent to $f(x,y) = e^x \cos(y)$ when $x = y = 0$. Use this to estimate the value of $\cos(0.2)$.

5. For $f(x,y) = x^2e^{-y}$, calculate $D_{\mathbf{u}}f(-2,0)$ when $\mathbf{u} = \langle 2, -3\rangle$.

6. Find the maximum rate of change of $f(x,y) = x^2y + \sqrt{y}$ at the point $(2,1)$.

7. Use a Riemann sum with $m = n = 3$ to approximate $\displaystyle\iint_R (xy^2 + x)\, dA$ where $R = [0,3] \times [2,8]$.

8. A contour map is shown for a function f on the square $R = [0,4] \times [0,4]$. Use A Riemann sum with midpoints (with $m = n = 2$) to estimate the value of $\displaystyle\iint_R f(x,y)\, dA$.



9. Calculate the exact value of $\displaystyle\iint_R (xy^2 + x)\, dA$ where $R = [0,3] \times [2,8]$.

10. Calculate the exact value of $\displaystyle\iint_R (xy^2 + x)\, dA$ where R is the triangle with vertices $(0,0)$, $(4,0)$, and $(4,2)$.