Power Series

1. Warm-up:

(a) What numerical values of x will make the series $\displaystyle\sum_{n=0}^{\infty} x^n$ converge?

☞ Would it be easier for you if we used the letter r instead of the letter x?

(b) For the values of x you found in (a), what is the *actual sum* of the series?

☞ Hopefully There should be an x in your answer, since the sum depends on what x is.

(c) What numerical values of x will make the series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$ converge?

(d) For the values of x you found in (c), what is the actual sum of the series?

2. Chloe: My personal favorite series is:

$$1 + 2x + 4x^2 + 8x^3 + 16x^4 + 32x^5 + \cdots$$

Can you help me write this in summation notation, please?

Group task: Help Chloe out.

Simon: Your series converges to $\dfrac{1}{1-2x}$.

Chloe: Thanks, Simon, but I think you forgot one detail.

Group chat: What values of x allow Chloe's series to converge? Is the sum really $\dfrac{1}{1-2x}$?

3. Find the sum of this series (in terms of x): $1 - x + x^2 - x^3 + x^4 - x^5 + \cdots$
Which values of x allow this series to converge?

4. Can you figure out what this series converges to and which values of x allow the series to converge?

$$1 + x^2 + x^4 + x^6 + x^8 + x^{10} + \cdots$$

5. Let's try the problem backwards now!

(a) Come up with a power series that converges to $\dfrac{1}{1-3x}$.

(b) Come up with a power series that converges to $\dfrac{1}{1-x^3}$.

(c) Come up with a power series that converges to $\dfrac{1}{4+x}$.

☞ Use algebra to make this look like $\frac{a}{1-r}$. Getting the "1" in place is the MOST important part.

(d) Come up with a power series that converges to $\dfrac{x^2}{1-x}$.

☞ Factor out x^2.

6. For what numerical values of x does each series converge? What is the radius of convergence for series?

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

☞ Cool fact: $0! = 1$.

(b) $\displaystyle\sum_{n=0}^{\infty} n!x^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n\,2^n}$

7. Experiment! Go back to #6(a) and plug in $x = 1$. Pull out a calculator/laptop and add up a TON of the terms. Discuss.