Partial Derivatives

1. Look up at the Wind Chill Function $w = f(T, v)$ on the screen. What is $w(20, 15)$?

Remember that T is "temperature" and v is "wind speed."

$w(20, 15) = 6$

2. Suppose that $T = \cancel{20}$ 15 and $v = 15$. Use the Wind Chill Function on the screen to answer the following:

(a) If $\Delta T = 5$, what is Δw?

$\Delta w = 6$

(b) If $\Delta T = -5$, what is Δw?

$\Delta w = -7$

(c) **Discuss:** What is your best guess for ΔW if $\Delta t = 1$? Why?

$\Delta T = 5$ corresponds to $\Delta w = 6$, so divide both by 5.

If $\Delta t = 1$, then Δw is about $\frac{6}{5}$.

(d) **Discuss:** What is your best guess for ΔW if $\Delta t = -1$? Why?

$\Delta T = -5$ corresponds to $\Delta w = -7$, so divide both by 5.

If $\Delta t = -1$, then Δw is about $-\frac{7}{5}$

(e) What do you think the derivative of w in the direction of T is?

Estimate the derivative: from part (c), $\frac{\Delta w}{\Delta T} = \frac{6}{5}$, and from part (d), $\frac{\Delta w}{\Delta T} = \frac{7}{5}$.

We could take the average and say the derivative is $\frac{1}{2}\left(\frac{6}{5} + \frac{7}{5}\right) = \frac{13}{10}$.

(f) If $\Delta v = 5$, what is Δw?

$\Delta w = -2$

(g) If $\Delta v = -5$, what is Δw?

$\Delta w = 3$

(h) **Discuss:** What is your best guess for ΔW if $\Delta v = 1$? Why?

If Δv is 1, then Δw is about $\frac{-2}{5}$.

(i) **Discuss:** What is your best guess for ΔW if $\Delta v = -1$? Why?

If Δv is -1, then Δw is about $\frac{3}{5}$.

(j) What do you think the derivative of w in the direction of v is?

From part (h), $\frac{\Delta w}{\Delta v} = -\frac{2}{5}$ From part (i), $\frac{\Delta w}{\Delta v} = \frac{-3}{5}$.

We could take the average and say the derivative is $\frac{1}{2}\left(\frac{-2}{5} - \frac{3}{5}\right) = \frac{1}{2}(-1) = \frac{-1}{2}$

3. Find the derivative of $f(x) = 2x^2 + 14$.

$$f'(x) = 4x$$

4. Find the derivative of $f(x) = 2x^2 + 14y$, pretending that y is a constant.

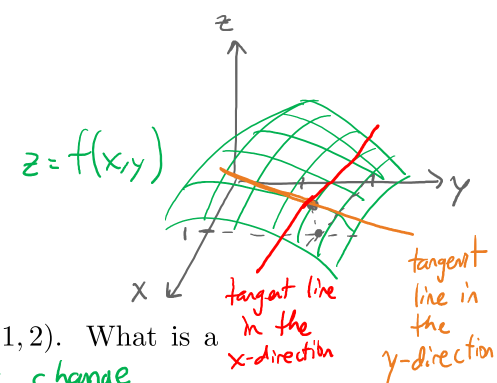
$$f'(x) = 4x$$

5. Now pretend that x is a constant and find the derivative of $f(y) = 2x^2 + 14y$.

$$f'(y) = 14$$

> **Notation Alert:**
> The partial derivative of f with respect to x is denoted f_x or $\dfrac{\partial f}{\partial x}$. ← x is the variable, anything else is constant
>
> The partial derivative of f with respect to y is denoted f_y or $\dfrac{\partial f}{\partial y}$. ← y is the variable, anything else is constant

6. Suppose that $f(x, y) = x^2 + y^2 + xy^2$.



(a) Find $f_x(x, y)$, the derivative of f with respect to x.

$$f_x = 2x + y^2$$

(b) Find $f_y(x, y)$, the derivative of f with respect to y.

$$f_y = 2y + 2xy$$

(c) Find $f_x(1, 2)$, the derivative of f with respect to x at the point $(1, 2)$. What is a graphical interpretation of this derivative?

$$f_x(1,2) = 2(1) + (2)^2 = 6$$

rate of change in the x-direction

(d) Find $f_y(1, 2)$, the derivative of f with respect to y at the point $(1, 2)$. What is a graphical interpretation of this derivative?

$$f_y(1,2) = 2(2) + 2(1)(2) = 8$$

rate of change in the y-direction

7. The *formula* for the windchill function is $w = 35.74 + 0.6215T - 35.75v^{0.16} + 0.4275Tv^{0.16}$. Use this to calculate the *exact* value of $\frac{\partial w}{\partial T}$ when $(T, v) = (20, 15)$.

Compare to your estimate from earlier!

$$\frac{\partial w}{\partial T} = 0.6215 + 0.4275\, v^{0.16}$$

$$\frac{\partial w}{\partial T}(20,15) = 0.6215 + 0.4275\,(15)^{0.16}$$

8. Repeat #6 with $f(x, y) = e^{x^2+y}$.

$$f_x(x,y) = \left(e^{x^2+y}\right)(2x) = 2x\,e^{x^2+y} \qquad f_x(1,2) = 2(1)\,e^{(1)^2+2} = 2e^3$$

$$f_y(x,y) = e^{x^2+y}(1) = e^{x^2+y} \qquad f_y(1,2) = e^{(1)^2+2} = e^3$$

9. Find each of the first partial derivatives of $f(x, y) = \cos(x^2 - y)$ at the point $(0, \pi)$.

$$f_x(x,y) = -\sin(x^2-y)(2x) \qquad f_x(0,\pi) = -2(0)\sin(0^2-\pi) = 0$$

$$f_y(x,y) = -\sin(x^2-y)(-1) \qquad f_y(0,\pi) = \sin(0^2-\pi) = 0$$