Lines and Planes

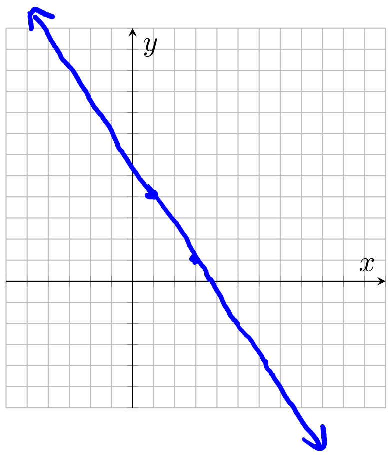
1. Group investigation: Pick several different values of t and plot the points whose x and y coordinates are given by:

👈 Everyone at your table can pick different values of t. Then share your answers.

$$\begin{cases} x = 1 + 2t \\ y = 4 - 3t \end{cases}$$

parametric equations → ; a point on the line (1,4) ; rate of change ; slope = $\frac{-3}{2}$

t is the parameter



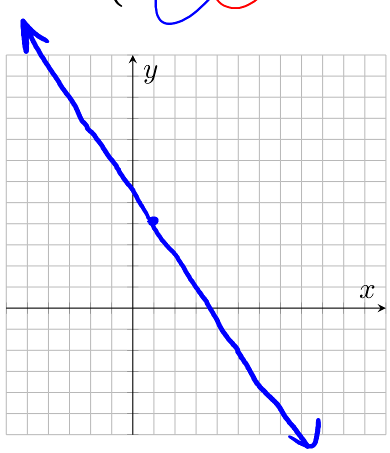
What shape results if you plot (x, y) for all values of t?

a line!

2. Another group investigation: Pick several different values of t and plot the points whose x and y coordinates are given by:

$$\begin{cases} x = 3 + 2t \\ y = 1 - 3t \end{cases}$$

point (3,1) ; slope = $\frac{-3}{2}$



What do you notice? it's the same line as in #1!

3. True or False: In two dimensions, a line is determined by:

(a) Two different points. TRUE

(b) One point and a slope. TRUE

(c) One point and a direction vector (i.e., a vector parallel to the line). TRUE

4. Write equations for the line passing through the point $(4, 7, -2)$ and parallel to the vector $\langle 3, -1, 2 \rangle$...

(a) In vector form:

$\langle x, y, z \rangle = \langle 4 + 3t, 7 - t, -2 + 2t \rangle$

$\langle x, y, z \rangle = \langle 4, 7, -2 \rangle + \langle 3, -1, 2 \rangle t$

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$ where $\vec{r}_0$ is an initial point and $\vec{v}$ is the direction vector

(b) In parametric form:

$$\begin{cases} x = 4 + 3t \\ y = 7 - t \\ z = -2 + 2t \end{cases}$$

5. True or False: In three dimensions, a *plane* is determined by:

☞ Make drawings, use your hands, use props, etc.!

(a) Three points (not all in the same line) TRUE

(b) One point and two different (non-parallel) direction vectors TRUE

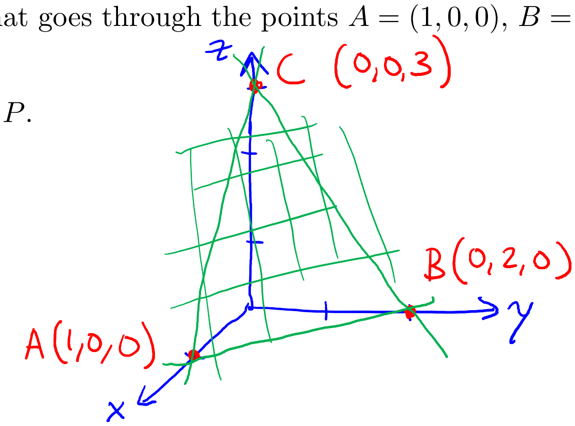
(c) One point and a perpendicular vector TRUE

"normal vector" to the plane

a vector perpendicular to the plane

6. Let's think about the plane P that goes through the points $A = (1, 0, 0)$, $B = (0, 2, 0)$, and $C = (0, 0, 3)$.

(a) Make a sketch of the plane P.



(b) One vector parallel to the plane is $\overrightarrow{AB} = \langle -1, 2, 0 \rangle$. A different vector parallel to P is $\overrightarrow{AC} = \langle -1, 0, 3 \rangle$. How can you find a vector $\mathbf{n}$ that is *perpendicular* to the plane P?

☞ Hint: You have TWO vectors that are parallel to the plane.

$\langle -1, 2, 0 \rangle \times \langle -1, 0, 3 \rangle = \langle 6, 3, 2 \rangle$ normal vector to the plane

$\vec{n} = \langle 6, 3, 2 \rangle$

(c) Suppose that $R = (x, y, z)$ is a random point located in plane P. Explain why the vector $\overrightarrow{RA} = \langle x - 1, y - 0, z - 0 \rangle$ is parallel to the plane P.

R and A are points in the plane

(d) We are going to find $\mathbf{n} \cdot \overrightarrow{RA}$ in two ways:

- Compute $\mathbf{n} \cdot \overrightarrow{RA}$ using the definition of the dot product.

$\vec{n} \cdot \overrightarrow{RA} = \langle 6, 3, 2 \rangle \cdot \langle x-1, y, z \rangle = 6(x-1) + 3y + 2z$

equal!

- Since $\overrightarrow{RA}$ is parallel to P and $\mathbf{n}$ is perpendicular to P, what is $\mathbf{n} \cdot \overrightarrow{RA}$?

Since $\vec{n}$ is perpendicular to $\overrightarrow{RA}$, then $\vec{n} \cdot \overrightarrow{RA} = 0$.

(e) Somebody just said, "WOW! That means an *equation* for the plane P is $6(x-1) + 3(y-0) + 2(z-0) = 0$." How did somebody arrive at this equation?

Set equal the things we found in part (d)

$6(x-1) + 3y + 2z = 0$ ← linear equation in 3D

(f) Explain why $6(x-0) + 3(y-2) + 2(z-0) = 0$ is another way to write an equation for plane P.

$\langle 6, 3, 2 \rangle \cdot \langle x-0, y-2, z-0 \rangle = 0$

normal vector — vector $\overrightarrow{RB}$

(g) What is the *general strategy* for finding the equation of a plane?

to be continued...

7. Find an equation for the plane that passes through the point $(2, 4, 3)$ and contains the line $\mathbf{r}(t) = \langle t, 2-t, 3+2t \rangle$.

8. *Spicy*: Find the angle between the planes $x + y + z = 1$ and $x - y + 3z = 3$.